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Spontaneous Lorentz Violation in QED and Standard Model - Physical and Astrophysical Consequences

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Abstract

We argue that the spontaneous Lorentz invariance violation (SLIV) provides massless vector Goldstone modes which may be successfully collected in the physical photon both in the properly extended Quantum Electrodynamics and Standard Model. However, in contrast to the spontaneous violation of internal symmetries, SLIV seems not to necessarily imply a physical breakdown of Lorentz invariance. Rather, when appearing in a gauge theory framework, like as QED and

SM, this may eventually result in a non-covariant gauge choice in an otherwise gauge invariant and Lorentz invariant theory. In substance the SLIV ansatz, due to which the vector field develops a vacuum expectation value (vev) $\langle A^{\mu}(x)\rangle = n^{\mu}M$ (where n^{μ} is a properly oriented unit Lorentz vector, while M is the proposed SLIV scale), may itself be treated as a pure gauge transformation with a gauge function linear in coordinates, $\omega(x) = (n^{\mu}x_{\mu})M$. In this sense, gauge invariance in QED leads to the conversion of SLIV into gauge degrees of freedom of the massless Goldstonic photon, unless it is disturbed by some extra gauge noninvariant terms. Actually, it appears in general that only an explicit breakdown of gauge invariance caused by these terms may allow to spontaneous Lorentz violation to manifest itself physically. These are the main issues, - namely, which could be the form of this gauge symmetry breaking both in QED and SM, and how this Lorentz invariant (by itself) breaking leads to the physical observation of spontaneous Lorentz violation - which we will address here. The presented Thesis is accordingly organized.

The first chapter contains a general discussion of the SLIV patterns and ways of their observation. Since the Lorentz superheavy Higgs component contributions are generally expected to be negligibly small at lower energies, one can, for more clearness and simplicity, completely exclude this Higgs component in the theory going to the nonlinear σ-model type SLIV for the vector field in the QED and SM theory. This procedure leads to a directly imposed vector field constraint $A_{\mu}^2 = n_{\mu}^2 M^2$, just as it takes place in the original nonlinear σ-model for pions. After considering some theoretical and experimental motivation for the Lorentz violation including recent OPERA data, its immediate consequences and experimental constraints, we formulate more precisely the sphere of our interests.

The second chapter is devoted to the SLIV mechanism which develops in the Abelian (QED type) theories with the nonlinear vector field constraint specially in the case when the internal U(1) charge symmetry is broken as well. We show that SLIV typically induces the genuine vector Goldstone boson which appears massless when the U(1) symmetry is exact and becomes massive in its broken phase. However, despite of the existence of Lorentz violating interactions, for both of phases an apparent Lorentz violation is completely canceled out in all the observable processes so that the physical Lorentz invariance in theory is ultimately restored.

In the third chapter alternative theory of the QED with non-exact gauge invariance is proposed. We show in this case that not only the SLIV photon appears as massless vector Nambu-Goldstone boson but also the Lorentz violation by itself, while being superficial in gauge invariant theory, becomes physically significant. This leads, through properly deformed dispersion relations appearing for charged fermions, to a new class of phenomena which could be of distinctive observational interest in particle physics and astrophysics. They include a significant change in the Greizen-Zatsepin-Kouzmin (GZK) cutoff for UHE cosmic ray nucleons, stability of high-energy pions and W bosons, modification of nucleon beta decays, and some others.

The fourth chapter discusses the extended Standard Model by terms which could follow from quantum gravity at very small distances. Arguing that exact gauge invariance may hide some generic features of SM, that could otherwise reveal themselves at high energies, the partial hypercharge gauge invariance is proposed. According to it, while the electroweak theory is basically $SU(2)\otimes U(1)_y$ gauge invariant being constructed from ordinary covariant derivatives of all fields involved, the $U(1)_y$ hypercharge gauge field B_μ field is allowed to form all possible polynomial couplings on its own and with other fields invariants. This unavoidably leads to the SLIV with the VEV being developed on some B field component, while its other components convert into the massless Nambu-Goldstone modes. After standard electroweak symmetry breaking they mix with a neutral W^3 boson of SU(2) that leads, as usual, to the massless photon and massive Z boson. Along with this the partial gauge invariance provides some distinctive SLIV effects in a laboratory mainly through slightly deformed dispersion relations of all SM fields involved. Being naturally suppressed at low energies these effects may become detectable in high energy physics and astrophysics. In contrast to the previous pure phenomenological studies, this theoretical approach allows to make in general the more definite predictions (or verify some earlier assumptions made ad hoc), and also discuss not only the time-like Lorentz violation but also the space-like case on which the current observational bounds appear to be much weaker.

And in the last **the fifth chapter** we collect our conclusions.

Keywords: *Origin of gauge symmetry; Spontaneous Lorentz violation; Constrained gauge fields; Gauge fields as Goldstone boson; Emergent gauge and gravity theories.*

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3. J.L. Chkareuli, Z. Kepuladze and G. Tatishvili, *Spontaneous Lorentz Violation via QED with Non-Exact Gauge Invariance*, European Physical Journal C 55 (2008) 309;
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Contents

Chapter 1

Introduction

1.1 Experimental and theoretical motivation

Lorentz invariance seems to be the cornerstone of modern particle physics and quantum field theory, and tends to be almost axiomatic. Farthermore, it also has wide application in modern technologies and affects its development, but one should not be harsh in conclusion and remember that Lorentz invariance manifests itself only with approximation following from experimental data, primarily from observations on high energy cosmic rays.

According to the present understanding of development of the universe, we see that it has tendency to cooldown from high symmetric phase to lower one, breaking almost all initially introduced internal symmetries. What if the same can be applied to the space-time symmetry? Initially exact space-time symmetry is broken at the lower scale and leads towards many new phenomena at high energies, but does not affect low energy physics being properly suppressed by small parameters defined by the scale of violation.

One may think that consideration of the violation of such fundamental symmetry as Lorentz symmetry is a wild idea but there is theoretical background suggesting such idea. Starting from the string theory where possibility of spontaneous Lorentz invariance violation was discovered long ago many other frameworks, such as modified gravity(loop gravity, etc.), have been investigated in this matter. Despite of the fact that all such high energy theories are very far from being directly discovered (due to their high scale), some times they can give a hint which in this case can be Lorentz invariance violation. Also there are many theories with modified QED or SM extensions, which contains all possible Lorentz violations and basically that leads to modification of dispersion relations of the particles involved, thus leading to many Lorentz violating effects in hadrons and leptons and gauge sector (via altering kinematics of the processes and some times even changing (or creating) the threshold energies of the corresponding reactions). However basically they do not provide effective dynamical framework and/or physical principles.

It should be admitted that apart from theoretical motivation there are experimental and observational hints, but because of the demanded extreme complexity and accuracy of such experiments and observations they are not very convincing and require farther investigations and improvements to be totally accepted or refused. First of all, it is worth to mention AGASA (Akeno Giant Air Shower Array) experimental group. They claimed 11 events of ultrahigh energy cosmic protons with energy more then $10^{20}eV$ were detected and that is odd for normal case since Greisen-Zatsepin-Kuzmin cut-off tells that high energy cosmic protons are loosing energy through scattering on the background relict radiation and no proton with energy $10^{20}eV$ or higher should be detected[1]. Usually, such observations have big resonance like the current case, many papers were trying to find explanation of this fact, but now picture tends to look like an experimental error.

There was also a work claiming Lorentz violation in high energy ions [1], but there is also another one 'contra-article' indicating mistake in Doppler effect calculation [2].

The latest is the observation the OPERA experiment announced (which is held in underground laboratory at Gran Sasso) [3]. Studying oscillation of μ-neutrinos, which were born and traveling from CERN they observed neutrinos traveling faster then light covering 730 km in 60 ns less time then the photon would have covered (OPERA group claims the accuracy of data to be 10%-15%), showing aberration from the speed of light in amount of $\delta = v_\nu^2 - 1 = 5 \cdot 10^{-5}$. Later experiment was repeated and it showed same results. This most evidently would be indication of Lorentz violation[2], but as it was said earlier even results were repeated and are compatible with the data of MINOS detectors [4] due to its extremely complexity and demanded high accuracy it is very early for any conclusion and farther investigation is required.

[1] Obviously, such fact, if found, may give an indication for modification of kinematics of the process. This could be caused, in turn, by modification of dispersion relation initiated by Lorentz violation.

[2] there are also the explanetions of this observation(using gravity(or gravity type modifications) for example) that preserves Lorentz invariance

1.2 Experimental restrictions on Lorentz invariance violation

Despite the fact that experimental motivation for the Lorentz violation is not very convincing, data which restricts Lorentz violation effects is very precise. Whatever origin Lorentz violation should not have, basically it is translated into the language of the modification of the dispersion relation, which without loosing generality can be written as:

$$p_\mu^2 = m^2 + \delta(E^2)E^2 \tag{1.1}$$

Form of $\delta(E^2)$ depends on the model. It may be constant or even have energy dependence with several energy regime. Considering case of constant δ parameter has its theoretical advantage, because such modification can be caused only by term with dimensionless parameter in front while in other case same parameter has to have some dimension (dimension depends on amount of derivatives) and by default that only means having problems with renormalizability, but even case of dimensionless parameter doesn't guarantee anything by itself.

Considering case of constant δ parameter one should remember that kinematics of the reaction is affected only if all participant particles do not have equal δ parameters. It is easy to prove since in any case you can introduce shifted four-momentum so to satisfy relation

$$p_\mu'^2 = m^2 \tag{1.2}$$

(p' is shifted four momentum and is linear respective to original one), weather it is time-like or space-like violation shifted momentum takes form

$$p'_\mu = p_\mu + \alpha(n^\nu p_\nu)n_\mu \tag{1.3}$$

here α should be found from given dispersion relation and n_μ is unit vector fixating preferred direction in the time-space as one could guess. In parallel, translation invariance still holds which means you have conservation law for shifted momentum as well

$$\sum P'_i = \sum P'_f \tag{1.4}$$

which directly means that all corresponding reactions' conditions still depend only on their pure masses.

For the framework of the constant δ there are several immediate and obvious restrictions. If the preferred frame of CMB is rotationally and translationally invariant then modification in the Lagrangian of the electromagnetic field of the type δB^2 (B is magnetic field and $\delta = |1 - c^2|$), δ is restricted in the following way: $\delta < 6 \cdot 10^{-22}$.

There are stringent restriction from so called vacuum Cerenkov radiation of the proton but it set on the difference of delta parameters. Assuming proton can have maximum attainable velocity greater then light it will start radiating photons and since proton with proven[3] maximum detected energy is 10^{20}eV, it constraints

$$\delta_p - \delta_\gamma < 10^{-23} \tag{1.5}$$

Same kind of restriction for the electron with maximum seen

[3] as it is mentioned above there are events from AGASA of protons with higher energy but those events need proper verification

cosmic energy 2 TeV is $\delta_e - \delta_\gamma < 10^{-13}$, but on other hand if the $\delta_e - \delta_\gamma < 0$, then photon have possibility to decay into electron-positron pair. Assuming that photon with detected highest energy 50 TeV is lower then threshold energy tells that

$$\delta_\gamma - \delta_e < 10^{-16} \tag{1.6}$$

If OPERA data is correct then it is good example of how $\delta(E^2)$ should have different behavior at different energies. OPERA suggests behavior like $\delta(E^2) = Const$ for neutrinos with energy about 20 GeV while observation of Supernova SN1987a, which by the way constraints at 10 MeV $\delta(E^2) \leq 10^{-9}$, with

$$\delta(E^2) = \left[\frac{E^2}{\mathcal{M}^2}\right]^n \tag{1.7}$$

favors $n \geq 2$ power dependence, but in some way there is inconsistency in OPERA data. Assuming time-like Lorentz violation for electron δ parameter can't be bigger then 10^{-13}, so for aberration $\delta = 5 \cdot 10^{-5}$ of neutrinos, they will start to lose their energy due to emitting electron-positron pair and hardly any neutrino should be able to reach Gran Sasso with detected energy, besides above said observations of atmospheric neutrinos with energies 1GeV-1TeV (by Super-Kamiokande) and 20-100 TeV (by IceCube collaboration) for the purpose of the absence of the same kind of electron-positron emission set limits $\delta < 10^{-8}$ and $\delta < 10^{-10} \div 10^{-11}$, which is few order stricter then SN1987a constraint [5].

There are also very strong constraints on flavor depended Lorentz violation. Assuming all neutrinos have own δ_i (i is a flavor index)parameters i.e. own maximal attainable velocities leads us to the mixing of the corresponding velocity states, thus to the oscillation. Assuming that the only origin of oscillation is velocity eigenstate mixing, the difference between velocities is constrained in the following way

$$|\delta_i - \delta_j| < 6 \cdot 10^{-22} \tag{1.8}$$

Also other constraints on velocity difference come from demand that if difference is too big it destroys coherence and erases the observed oscillation dips, therefore [6]

$$|\delta_i - \delta_j| < 10^{-19} \div 10^{-21} \tag{1.9}$$

but as we see they rest comparing to previous one [4].

Even in all considered cases constraints on Lorentz symmetry violating effects are very strict, the theories with Lorentz violation put in by hand can easily satisfy all experimental and observational data by tuning Lorentz breaking parameters. So, without physical principle it is very hard to choose a way of violation. Fortunately there is experience that teaches that spontaneous symmetry breaking is a fancy way and also a tool allowing to hold the ties of the theory and do not lose control over it, speaking of which should be admitted that despite of all the fanciness and helpfulness of this approach Nambu-Jona-Lasinio pion is the only Goldstone boson found so far in the particle physics. Anyway, it is very compelling to consider every massless field as a Goldstone boson especially

[4] Some of these phenomena and some others are duscussed later in framework of the developed theories.

vector bosons or tensor boson which can be identified with photon, Yang-Mills vector field or the graviton and the masslessness will be guaranteed by their Goldstonic nature and the gauge invariance will be invoked by symmetry violation.

1.3 Spontaneous Lorentz invariance violation(SLIV)

Accepting idea of spontaneous symmetry breaking respective to Lorentz invariance we need to guarantee degenerate vacuum state in order to allow spontaneous Lorentz invariance violation. In quantum electrodynamic(QED) framework vector field's self interaction potential terms which satisfy given global and Lorentz symmetry will give a degenerate vacuum

$$L = -\frac{1}{4}F_{\mu\nu}^2 + \overline{\Psi}(i\gamma D + m)\Psi + V(A_\lambda^2) \tag{1.10}$$

where

$$V(A_\mu^2) = \frac{\lambda^2}{4}(A_\mu^2 - n_\mu^2 M^2)^2 \tag{1.11}$$

and n_μ is unit(time-like or space-like) vector defining some preferred direction in space-time. Such potential with degenerate vacuum states with minimum in $n_\mu^2 M^2$ leads to condensation of A_μ field with vacuum expectation value(VEV) M toward direction of the n_μ and no matter is the n_μ time-like or space-like, just in time-like case VEV will evolve on time component A_0 and at same time break $SO(1,3)$ Lorentz symmetry to $SO(3)$ rotation symmetry and in space-like case VEV evolves on one of the spatial component breaking Lorentz symmetry $SO(1,3)$ to the "reduced Lorentz symmetry" $SO(1,2)$. As a consequence of the spontaneous symmetry breaking we fix axial gauge for goldstone vector mode $n_\mu a^\mu = 0$ (a^μ is vector Goldstone mode)[5], which will be identified with the photon.

Such framework is particularly interesting because it can generate on its own non-linear constraint on vector field A_μ, which can be easily shown taking derivative from the Euler-Lagrange equation

$$\partial_\nu(\partial_\mu F^{\mu\nu} - j^\nu) = \lambda(A_\mu^2 - n_\mu^2 M^2)\partial_\nu A^\nu + \lambda A^\nu \partial_\nu(A_\mu^2 - n_\mu^2 M^2) \tag{1.12}$$

j^ν is the matter current which conserves and therefore left hand side is equal to zero, leaving us with the solution from right hand side

$$A_\mu^2 - n_\mu^2 M^2 = 0 \tag{1.13}$$

In parallel taking limit $\lambda \to \infty$ and having M finite we go from linear model to the nonlinear one with a given constraint (1.13). This constraint is the condition of SLIV , but what it does is little more. It excludes Higgs mode from theory considering only vector goldstone mode and therefore form "nonlinear sigma model" (expressed in terms of Goldstonic mode), because it means that Higgs mode of the Lorentz symmetry violation is infinitely massive since its mass is proportional to taken to infinity λM as a result leaving no chance for Higgs mode to anyhow affect theory. Similar model had been considered long ago by Nambu [7]. Not interested in the dynamical mechanism causing spontaneous Lorentz violation successfully using technique of nonlinear symmetry realization he showed in the tree approximation and only for time-like violation that non-linear constraint (1.13) appears in fact as a possible gauge

[5] details of the expansions in goldstone mode will be discused in the next chapter

choice at huge SLIV scale M. This result has been extended to the one-loop approximation [8] and for both the time-like ($n_\mu^2 > 0$) and space-like ($n_\mu^2 < 0$) Lorentz violations.

This model is essentially nonlinear and contains Lorentz violating vertexes, but that does not change a fact that Lorentz invariance breaking is superficial. All Lorentz violating effects, all the contributions to the photon-photon, photon-fermion and fermion-fermion interactions violating the physical Lorentz invariance happen to be exactly cancelled with each other in the manner observed by Nambu a long ago for the simplest tree-order diagrams leaving S-matrix unaltered and thus making them physically unobservable. These results hold even when quantum effects are taken into consideration at least in one loop approximation. Reason behind this unobservability is initially involved gauge invariance, same gauge invariance allows all Goldstonic modes to form together a vector goldstone mode. Non-linear constraint serves as a nonlinear gauge choice and theory itself is physically indistinguishable from the ordinary one in axial gauge.

1.4 The plan

Consideration of the SLIV in the framework of QED with exact and non-exact gauge invariance shows special role which gauge invariance has [9, 10], which obviously is in close touch with corresponding internal symmetry. Special properties of the global and local internal symmetries with respect SLIV, their influence on observability of the Lorentz violation and on the type of Lorentz breaking effects and the actual physical Lorentz violation in the realistic model are the topic of the investigation of this thesis.

Chapter 2

Generation of massless and massive vector boson

2.1 The model

Lorentz invariance spontaneous violation could generally cause the occurrence of the corresponding massless Nambu-Goldstone modes which are believed to be photons or other gauge fields. The effective theoretical laboratory for the SLIV consideration happens to be some simple class of the QED type models for the starting vector field A_μ. Therefore we consider here the spontaneous Lorentz violation in the framework of QED with standard Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma\partial + m)\psi - eA_\mu\overline{\psi}\gamma^\mu\psi \tag{2.1}$$

with the dynamically appeared nonlinear four-vector field constraint (1.13) in the case when the internal $U(1)$ charge symmetry is also spontaneously broken so that the massless vector Goldstone boson (photon) having been generated through the SLIV becomes then massive in the $U(1)$ symmetry Higgs phase. For this purpose one needs to extend the starting Lagrangian L_{QED} (2.1) by the scalar field part

$$\mathcal{L}(\varphi) = |D_{\mu}\varphi|^2 - m_{\varphi}^2\varphi^*\varphi - \frac{\lambda_{\varphi}}{2}(\varphi^*\varphi)^2 \tag{2.2}$$

where $D_{\mu}\varphi = (\partial_{\mu} - ieA_{\mu})\varphi$ is a standard covariant derivative for the charged scalar field φ from which the above Goldstonic photon gets its mass. It will be displayed again that the apparent Lorentz violation caused by the nonlinear SLIV constraint (1.13) is completely canceled out in all the physical processes in the same manner as in the massless QED case considered earlier [7, 8].

We consider first the massive non-linear QED Lagrangian (Sec.2) appeared once the dynamical constraint (1.13) is explicitly implemented into Lagrangians (2.1, 2.2) and internal $U(1)$ symmetry is spontaneously broken so that the photon becomes massive. We derive the general Feynman rules for the basic photon-photon and photon-fermion interactions, as well as the rules related with Higgs sector of theory. The model appears in essence two-parametric containing the electric charge e and inverse SLIV scale $1/M$ as the perturbation parameters so that the SLIV interactions are always proportional some powers of them. Then in Sec.3 various SLIV processes such the massive photon scattering off the charged fermion, Higgs boson decays and photon-photon scattering are considered in detail. All these effects, both in the tree and one-loop approximation, appear in fact vanishing so that the physical Lorentz invariance is ultimately restored. And, finally, we give our conclusions in Sec.4.

2.2 The Lagrangian and Feynman rules

2.2.1 The Lagrangian: U(1) symmetry phase

We consider simultaneously both of the above-mentioned SLIV cases, time-like or space-like, introducing some properly oriented unit Lorentz vector n_μ $(n_\mu^2 \equiv n^2 = \pm 1)$ so as to have the following general parametrization for the vector potential A_μ in the Lagrangian (2.1) of the type

$$A_\mu^2 = n^2 M^2, \quad A_\mu = a_\mu + n_\mu (n \cdot A) \tag{2.3}$$

(hereafter M^2 is defined strictly positive) where the a_μ is pure Goldstonic mode

$$n \cdot a = 0 \tag{2.4}$$

while the Higgs mode (or the A_μ component in the vacuum direction) is given by the scalar product $n \cdot A$. Substituting this parametrization into the vector field constraint (1.13) one comes to the equation for $n \cdot A$ (taking, for simplicity, the positive sign for the square root and expanding it in powers of $\frac{a_\nu^2}{M^2}$)

$$n \cdot A \ = \left[(M^2 - n^2 a_\nu^2)\right]^{\frac{1}{2}} = M - \frac{n^2 a_\nu^2}{2M} + O(1/M^2) \tag{2.5}$$

We proceed further putting that new parametrization (2.3) into our basic Lagrangians (2.1) and (2.2), then expand it in powers of $\frac{a_\nu^2}{M^2}$ and make the appropriate redefinition of the fermion and scalar fields according to

$$\psi \rightarrow e^{ieM(n\cdot x)}\psi, \quad \varphi \rightarrow e^{ieM(n\cdot x)}\varphi \tag{2.6}$$

so that the bilinear fermion and scalar terms, $eM\overline{\psi}(\gamma \cdot n)\psi$ and $\varphi^*[ieM(\overleftrightarrow{\partial} \cdot n) + e^2n^2M^2]\varphi$, appearing, respectively, from the expansion of the fermion and charged scalar current interactions in the Lagrangians (2.1, 2.2) are exactly cancelled by an analogous terms stemming now from their kinetic terms (the abbreviation $\overleftrightarrow{\partial}$ means, as usual, $\varphi^*\overleftrightarrow{\partial}\varphi = \varphi^*(\partial\varphi) - (\partial\varphi^*)\varphi$). So, we eventually arrive at the nonlinear SLIV Lagrangian for the Goldstonic a_μ field (denoting its strength tensor by $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$)

$$\begin{aligned}\mathcal{L}(a,\psi,\varphi) &= -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}\lambda(n\cdot a)^2 - \frac{n^2}{4M}f_{\mu\nu}\left[(n^\mu\partial^\nu - n^\nu\partial^\mu)\,a_\rho^2\right] + \\ &+\overline{\psi}(i\gamma\partial + m)\psi - ea_\mu\overline{\psi}\gamma^\mu\psi + \frac{en^2a_\rho^2}{2M}\overline{\psi}(\gamma\cdot n)\psi + \\ &+|(\partial_\mu - iea_\mu)\varphi|^2 - \frac{ien^2a_\rho^2}{2M}[\varphi^*(\overleftrightarrow{\partial}\cdot n)\varphi] \; - P(\varphi)\end{aligned} \tag{2.7}$$

explicitly including its orthogonality condition $n \cdot a = 0$ through the term which can be treated as the gauge fixing term (taking the limit $\lambda \rightarrow \infty$). Note that there are presented only the terms of the first order in $\frac{a_\nu^2}{M^2}$ in the Lagrangian and also retained the former notations for the fermion ψ and scalar field φ (with its unchanged potential $P(\varphi)$ included, as is given in the starting Lagrangian (2.2)).

The Lagrangian (2.7) completes the nonlinear σ model type construction for quantum electrodynamics for the charged fermion and scalar fields . The model contains the massless vector Goldstone boson modes (keeping the massive Higgs mode frozen), and in the limit $M \to \infty$ is indistinguishable from conventional QED taken in the general axial (temporal or pure axial) gauge. So, for this part of the Lagrangian $L(a, \psi, \varphi)$ given by the zero-order terms in $1/M$ the spontaneous Lorentz violation only means the noncovariant gauge choice in otherwise the gauge invariant (and Lorentz invariant) theory. Remarkably, furthermore, also all the other terms in the $L(a, \psi, \varphi)$ (2.7), though being by themselves the Lorentz and $C(CPT)$ violating ones, cause no the physical SLIV effects which appear strictly cancelled in all the physical processes involved. As shows the explicit calculations, there is a full equivalence of such a model with a conventional quantum electrodynamics at least in the tree [7] and one-loop[8] approximation taken for the pure fermionic part in the Lagrangian (2.7). The same conclusion can obviously be expected for its scalar part as well. This seems to confirm that the starting vector field condition $A_\mu^2 = n^2 M^2$ which results in the nonlinear QED model (2.7) is the gauge choice rather non-trivial dynamical constraint which might come to the physical Lorentz violation.

2.2.2 The Lagrangian: U(1) symmetry broken phase

Let us now turn to the case of the spontaneous Lorentz violation when the accompanying internal $U(1)$ symmetry in the SLIV Lagrangian $L(a, \varphi)$ (2.7) is also spontaneously broken. For this purpose one replaces, as usual, the scalar mass squared $m_\varphi^2 \to -m_\varphi^2$ in its potential $P(\varphi)$ so that the scalar φ now develops the VEV

$$\varphi = \frac{1}{\sqrt{2}}(\eta(x) + v)e^{i\xi(x)/v}, \; v^2 = 2m_\varphi^2/\lambda_\varphi \tag{2.8}$$

where for the scalar field φ the standard polar parametrization is used with the proper Higgs and Goldstone modes, $\eta(x)$ and $\xi(x)$, involved. Putting the shifted scalar field (2.8) into the Lagrangian (2.7) one comes to the final SLIV theory with the broken $U(1)$ symmetry

$$\begin{aligned}\mathcal{L}(a,\psi,\eta,\xi) &= \mathcal{L}(a,\psi)+\frac{1}{2}(\partial_\mu\eta)^2+\frac{1}{2}(\eta+v)^2[\partial_\mu(\xi/v)-ea_\mu]^2+\\ &+\frac{en^2a_\rho^2}{2M}(\eta+v)^2(\partial\cdot n)(\xi/v)+P(\eta)\end{aligned} \tag{2.9}$$

where $L(a,\psi)$ stands for the vector field and fermion part (both linear and nonlinear) as is given in the Lagrangian $L(a,\psi,\varphi)$ (2.7), while $P(\eta)$ denotes an ordinary polynomial of the scalar Higgs component η appeared. One can see that the vector Goldstone boson a_μ acquires the mass term $\frac{1}{2}(e^2v^2)a_\mu^2$. However, apart from that, there appears the scalar-vector (goldston-goldston) mixing term in the Lagrangian $L(a,\psi,\eta,\xi)$. In an ordinary Higgs mechanism case such a mixing term can easily be removed by choosing a proper unitary gauge. However, it is hardly possible in the SLIV case where, as is seen from the above Lagrangian $L(a,\psi,\varphi)$ (2.7), one has already come to the axial gauge choice for the vector Goldstonic boson a_μ once the spontaneous Lorentz violation occurred. So, one may not put now extra (unitary) gauge to get rid of the scalar Goldstone field $\xi(x)$. Nonetheless, this field, if it were appeared in the theory, would correspond to the unphysical particle in the sense that it could not appear as incoming or outgoing lines in Feynman graphs, as was recently argued [11] in the context of Standard Model taken in the axial gauge. This can be seen at once by diagonalizing the bilinear $a-\xi$ mixing term in our Lagrangian $L(a,\psi,\eta,\xi)$ (2.9) so that the ξ field disappears in it, while leading to the more complicated form for the a boson gauge fixing term. In this connection, the option of an existence of the starting ξ field in the Lagrangian (2.9) , while having in momentum space the diagonalized a and ξ propagators, happens to be more convenient and transparent and we take this way in what follows.

2.2.3 The Feynman rules

Actually, rewriting the $a-\xi$ mixing term in the Lagrangian $L(a,\psi,\eta,\xi)$ (2.9) in momentum space and diagonalizing it by the substitution

$$\xi(k) \to \xi(k) + i\mu \frac{k^\mu a_\mu(k)}{k^2} \tag{2.10}$$

where $\mu = ev$ is the vector a boson mass, one has for this term

$$\frac{1}{2}(e\eta/\mu+1)^2 \left[-ik^\mu \xi(k) + \mu\left(\frac{k^\mu k^\nu}{k^2} - g^{\mu\nu}\right) a_\nu(k)\right]^2 \tag{2.11}$$

with the new pure $\xi(k)$ and $a_\mu(k)$ states appeared. Note that just this transversal bilinear form for the a boson in (2.11) together with its kinetic terms and the gauge fixing condition in the Lagrangian $L(a,\psi,\varphi)$ (2.7) determines eventually the diagonalized propagator for the massive a boson of the type (in the limit $\lambda \to \infty$)

$$D^{(a)}_{\mu\nu}(k) = \frac{-i}{k^2-\mu^2+i\epsilon}\left(g_{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{n\cdot k} + \frac{n^2 k_\mu k_\nu}{(n\cdot k)^2}\right) \tag{2.12}$$

whose numerator is happened to be the same as for the axially gauged massless vector boson. Meanwhile the propagator for the massless scalar field ξ amounts to

$$D^{(\xi)}(k) = \frac{i}{k^2} \tag{2.13}$$

For the vector boson a_μ being orthogonal to n^μ, one can choose a basis of two transverse (in momentum space) components $\epsilon_\mu^{(t)}(k)$ $(t = 1, 2)$

$$n^\mu \epsilon_\mu^{(t)}(k) = 0, \quad k^\mu \epsilon_\mu^{(t)}(k) = 0 \tag{2.14}$$

and the 'preferred' component $\epsilon_\mu^{(n)}(k)$ determined by the particular SLIV direction n^μ

$$\epsilon_\mu^{(n)}(k) = N(k_\mu - n_\mu \frac{n \cdot k}{n^2}), \quad n^\mu \epsilon_\mu^{(n)}(k) = 0, \quad k^\mu \epsilon_\mu^{(n)}(k) = N(\mu^2 - \frac{(n \cdot k)^2}{n^2}) \tag{2.15}$$

where the normalization factor N is proposed to be chosen in such way that the sum of all the polarizations amounts to the numerator of the a boson propagator (2.12).

Supplementing this propagator and ordinary Feynman rules by the rules concerning the Lorentz violating interactions (see also[8]) in the Lagrangians $L(a, \psi, \eta, \xi)$ (2.9) and $L(a, \psi, \varphi)$ (2.7), particularly, those for the contact a^2-fermion vertex

$$i\frac{e g_{\mu\nu} n^2}{M}(\gamma \cdot n) \tag{2.16}$$

and the a^3 vertex (rewriting it first as the $-\frac{n^2}{M}(\partial_\mu a_\nu n^\mu a_\rho \partial^\nu a^\rho)$)

$$-\frac{in^2}{M}\left[(k_1 \cdot n)k_{1,\alpha} g_{\beta\gamma} + (k_2 \cdot n)k_{2,\beta} g_{\alpha\gamma} + (k_3 \cdot n)k_{3,\gamma} g_{\alpha\beta}\right] \tag{2.17}$$

(where the second index in the each photon 4-momentum k_1, k_2 and

k_3 denotes its Lorentz component) one is ready to calculate some of the low-order (in $1/M$) processes related with the a boson and fermion. Note that the scalar field ξ is not coupled to fermions and, therefore, is not considered in the the a-boson-fermion interactions. However, one should include into consideration another a^3 vertex which appears from the $a^2-\xi$ coupling in the final Lagrangian $L(a,\psi,\eta,\xi)$ (2.9) once the $a-\xi$ diagonalization (2.10) in momentum space has been carried out:

$$\frac{in^2\mu^2}{M}\left[\frac{(k_1\cdot n)}{k_1^2}k_{1,\alpha}g_{\beta\gamma}+\frac{(k_2\cdot n)}{k_2^2}k_{2,\beta}g_{\alpha\gamma}+\frac{(k_3\cdot n)}{k_3^2}k_{3,\gamma}g_{\alpha\beta}\right] \tag{2.18}$$

One can see that for the a bosons being on the mass shell, $k_{1,2,3}^2=\mu^2$, the vertices (2.17) and (2.18) exactly cancel each other.

The other rules related with interactions of scalar Higgs and Goldstone fields, η and ξ, will be given in the next section.

2.3 SLIV processes in massive QED

We show now by a direct calculation of the tree level amplitude for Compton scattering of the massive vector Goldstone a boson off the charged fermion and other processes that the spontaneous Lorentz violation, being superficial in the massless nonlinear QED with an exact $U(1)$ symmetry involved [7, 8], is still left hidden even though this symmetry is spontaneously broken and the photon is getting mass.

2.3.1 Vector boson scattering on fermion

The Lorentz violating part of the elastic a-boson-fermion scattering is, as follows from the Lagrangians $L(a, \psi, \eta, \xi)$ (2.9) and $L(a, \psi, \varphi)$ (2.7)), the only SLIV fermionic process which appears in the lowest $1/M$ order. This process is concerned with two diagrams one of which is given by the direct contact a^2-fermion vertex (2.16), while another corresponds to the a boson exchange induced by the a^3 couplings (2.17) and (2.18). Owing to the above-mentioned mutual cancellation of these a^3 couplings for the on-shell a bosons, only the third terms in them contributes in the case considered so that one comes to the simple matrix element iM for the these two diagrams

$$i\mathcal{M} = i\frac{en^2}{M}\bar{u}(p_2)\left[(\gamma \cdot n) + i(1 - \frac{\mu^2}{k^2})(kn)k_\beta\gamma_\alpha D^{(a)}_{\alpha\beta}(k)\right]u(p_1) \cdot [\epsilon(k_1) \cdot \epsilon(k_2)] \quad (2.19)$$

where the spinors $u(p_{1,2})$ and polarization vectors $\epsilon(k_{1,2})$ stand for the ingoing and outgoing fermions and a bosons, respectively, while k is the 4-momentum transfer $k = p_2 - p_1 = k_1 - k_2$. After further simplifications in the square bracket related with the explicit form of the a boson propagator $D^{(a)}_{\alpha\beta}(k)$ (2.12) and the fermion current conservation $\bar{u}(p_2)(\hat{p}_2 - \hat{p}_1)u(p_1) = 0$, one is finally led to the total cancellation of the Lorentz violating contributions to the Compton scattering of the massive vector Goldstone boson a

$$i\mathcal{M}_{SLIV}(a + \psi \to a + \psi) = 0 \quad (2.20)$$

One could say that such a result may be in some sense expected since from the SLIV point of view the massive QED which we considered here is hardly differed from the massless one[7]. Actually, the fermion current conservation, which happens crucial for the above cancellation, works in both of cases depending no whether the internal $U(1)$ symmetry is exact or spontaneously broken. The fermion sector (being no coupled to the charged scalar from the outset (2.7)) still possesses this symmetry at least in tree level approximation thus leading to the SLIV cancellation.

2.3.2 Higgs boson decays

Remarkably, the situation is not changed in the Higgs sector where the $U(1)$ symmetry related with the starting charged scalar field seems to be directly broken and, therefore, the physical SLIV might appear. Let us examine, for sure, the Lorentz violating Higgs boson decay $\eta \to 3a$ which also appears in the lowest $1/M$ order if the masses of the η and a bosons are properly arranged, $m_\eta > 3\mu$ (or $e < \sqrt{2\lambda_\varphi/9}$ according to Eq.(2.8)).

As one can see from the Lagrangian $L(a, \psi, \eta, \xi)$ (2.9) with the substitution (2.10) already made, this decay goes through the contact $\eta - a^3$ coupling leading to the matrix element

$$i\mathcal{M}_{cont} = i\frac{en^2}{M\mu}\eta(k)\sum_{l,m,n} P^{lmn}[\epsilon(k_m)\cdot\epsilon(k_n)][k_l\cdot\epsilon(k_l)](k_l\cdot n) \tag{2.21}$$

where the external 4-momenta $k_{l,m,n}$ $(l, m, n = 1, 2, 3)$ of all three a-bosons with the polarization vectors $\epsilon(k_{l,m,n})$ are supposed to be picked up according the symmetrical projection operator P^{lmn} $(l, m, n = 1, 2, 3)$ introduced which takes the nonzero value 1 for only the non-equal index values $(l \neq m \neq n)$, and also the on-shell condition $(k_{l,m,n})^2_\alpha = \mu^2$ has been used; furthermore, $\eta(k)$ stands for the Higgs boson wave function and the total energy-momentum conservation is supposed, $k = k_l + k_m + k_n$.

Apart from this contact diagram, the $\eta \to 3a$ decay stems via the pole diagrams corresponding to the intermediate a and ξ boson exchange. They are diagrams where the η decays first into two a bosons or into a and ξ bosons (with momenta k_1 and k_2) due to the normal Lorentz invariant vertexes stemming from Eq.(2.11)

$$2ie\mu \left(\frac{k_1^\mu k_1^\nu}{k_1^2} - g^{\mu\nu}\right)\left(\frac{k_2^\mu k_2^\rho}{k_2^2} - g^{\mu\rho}\right) \tag{2.22}$$

$$2ek_1^\mu \left(\frac{k_2^\mu k_2^\rho}{k_2^2} - g^{\mu\rho}\right) \tag{2.23}$$

followed then by the virtual Lorentz violating transitions $a \to 2a$ and $\xi \to 2a$ given, respectively, by the a^3 couplings (2.17,2.18) and by the $a^2 - \xi$ vertex in the Lagrangian (2.9)

$$\frac{n^2\mu}{M}(n \cdot k)g^{\mu\nu} \tag{2.24}$$

These six pole diagrams (three diagrams for the each type exchange) correspond, respectively, to the cases when one of a bosons with 4-momentum k_l $(l = 1;2;3)$ is produced directly, whereas two other a bosons with momenta k_m and k_n $(m,n = 2,3;1,3;1,2)$ appear from the virtual a and ξ boson.

Using the above projection operator P^{lmn} one can calculate the decay amplitude according to all these pole diagrams simultaneously. Note that that all the momenta in the above Feynman rules are measured ingoing so that for the outgoing ξ state the vertexes (2.23) and (2.24) should get a minus sign. Again, owing to the already mentioned mutual cancellation of the a^3 vertices (2.17) and

(2.18) for the on-shell a bosons, only one of their terms contributes in the a boson exchange diagrams. Remarkably, the non-pole contribution in these a boson exchange terms appears to be completely cancelled (when gauge fixing condition $n \cdot \epsilon(k_{l,m,n}) = 0$ is used) with the contact diagram contribution iM_{cont} (2.12) , while the the pole contribution terms are happened to be exactly cancelled with the terms stemming from the intermediate ξ boson diagrams. So, one eventually has that the total amplitude for the Lorentz-violating $\eta \to 3a$ decay is certainly vanished

$$i\mathcal{M}_{SLIV}(\eta \to 3a) = 0 \tag{2.25}$$

2.3.3 Other processes

In the next $1/M^2$ order the Lorentz violating $a - a$ scattering is also appeared. Its amplitude is concerned with the a boson exchange diagram and the contact a^4 interaction diagram following from the higher terms in $\frac{a_\nu^2}{M^2}$ in the Lagrangian (2.7). Again, these two diagrams are exactly cancelled giving no the physical Lorentz violating contributions.

The same conclusion seems to be derived for the higher order processes including both the tree diagrams and the loops concerning the a bosons and fermions. Actually, as in the massless QED case considered earlier [8], the corresponding one-loop matrix elements, when they do not vanish by themselves, amount to the differences between pairs of the similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of the external four-momenta of the particles involved) that in the framework of the dimensional regularization leads to their total cancellation.

Chapter 3

QED with non-exact gauge invariance

3.1 The Model

It is now generally accepted that internal gauge symmetries form the basis of the modern particle physics being most successfully realized within the celebrated Standard Model of quarks and leptons and their fundamental strong, weak and electromagnetic interactions.

However, as was discussed time and again (see, for an example, [12]), local gauge symmetries, unlike global symmetries, represent redundancies of the description of a theory rather than being "true" symmetries. Indeed, the very existence of gauge invariance means that there are more field variables in the theory than are physically necessary. Usually, these superfluous degrees of freedom are eliminated by some gauge-fixing conditions which have no a special physical sense by themselves and actually are put by hand. Instead, one could think that these extra variables would vary arbitrarily with time so that they could be made to serve in description of some new physics.

In this connection, one of the most interesting example seems to be the spontaneous Lorentz invariance violation (SLIV) phenomenon which may actually be hidden by an exact gauge invariance. Indeed, while the first models realizing the SLIV conjecture were based on the four fermion interaction [13], the simplest model for SLIV is in fact given by a conventional QED type Lagrangian extended by an arbitrary vector field potential energy

$$U(A) = \frac{\lambda}{4}\left(A_\mu A^\mu - n^2 M^2\right)^2 \tag{3.1}$$

which is obviously forbidden by a strict gauge invariance. So, if one could moderate the exact gauge invariance requirement in a way to allow the vector field potential energy (3.1) to be included into the conventional QED Lagrangian, then the time-like or space-like SLIV could unavoidably hold. Expanding the vector field around vacuum configuration,

$$A_\mu(x) = n_\mu(M+\phi) + a_\mu(x)\ , \quad n_\mu a^\mu = 0 \tag{3.2}$$

one finds that the a_μ field components, which are orthogonal to the Lorentz violating direction n_μ, describe a massless vector Nambu-Goldstone (NG) boson, while the $\phi(x)$ field corresponds to a Higgs mode. This minimal polynomial QED extension, being sometimes referred to as the "bumblebee" model is in fact the well-known prototype SLIV model intensively discussed above and in the literature (see [14] and references therein).

This argumentation allows to think that the SLIV pattern according to which just the vector field (rather than some scalar field derivative [15] or vector field stress-tensor [16]) develops the vacuum expectation value (VEV) could require the gauge principle to be properly weakened. While the ultimate goal is to consider SM symmetries in this connection for the good guideline is proposed extension of a conventional QED to the higher dimension couplings included. We are reminded that gauge invariance in a standard quantum electrodynamics is not necessarily postulated for the photon-fermion interaction - it appears on its own if, apart from relativistic invariance, the restrictions related with the conservation of parity, charge-conjugation symmetry and number of fermions are also imposed in the Lagrangian. Actually, one uses gauge invariance only if one constructs the photon kinetic term to have an ordinary $F_{\mu\nu}F^{\mu\nu}$ form since this is necessary in order that the Hamiltonian be bounded below[1]. Similarly, analogous restrictions for photon-fermion couplings of higher dimensions generally allow only for a few new ones (for each order in the theory's inverse scale $1/M$) which appear to possess, however, some approximate gauge invariance rather than an exact one as one has in a conventional QED with dimensionless coupling constants. In this connection the most transparent situation arises in a minimal QED extension to dimension-five couplings which we consider here in detail. Since this extension, apart from photon-fermion interaction terms,

[1] Note also that a general photon kinetic term gives rise to ghosts in the propagator and, specifically in the Lorentz-violating QED type theory, to a domain wall solution for the vector potential A_μ that might lead to a wall-dominated early Universe and its immediate collapse [32].

will necessarily include the free fermion bilinear of type $(1/M)\partial_\mu\overline{\psi}\partial^\mu\psi$ one could hold to the idea that free fermions would generally be described by some combined Dirac-Klein-Gordon equation rather than the pure Dirac equation that might be hidden at low energies. However, due to spontaneous Lorentz violation this "fermion-boson complementarity" could become significant providing a somewhat natural model for a tiny gauge non-invariance in the QED when the electromagnetic interaction is "switched on". As a result, the SLIV, having been superficial in gauge invariant theory, becomes in fact physically observable through a certain dispersion relation which automatically appears for charged fermions. This is in contrast to the direct Lorentz violation models [17, 18] where some modified dispersion relations for the photon and/or matter particles involved are in essence specially postulated.

One can start with a free Lagrangian for some massive charged fermion ψ in the form

$$L(\psi) = \overline{\psi}(i\gamma_\mu\partial^\mu - m_0)\psi + \frac{1}{\mathcal{M}}\partial_\mu\overline{\psi}\cdot\partial^\mu\psi \tag{3.3}$$

which contains, apart from a true fermionic kinetic term, some "bosonic" type kinetic term as well. As is clear from this Lagrangian, the fermion dispersion relation will be a little changed so that for its four-momentum p_μ squared one has

$$p^2 = (m_0 - p^2/\mathcal{M})^2 = m_0^2(1 - 2m_0/\mathcal{M} + \cdots) \tag{3.4}$$

which leads to a tiny mass shift for fermion which, of course, is of no experimental interest. Let us now turn on all possible interaction terms which, under the foregoing discrete and global symmetry restrictions taken, amount to the gauge type "minimal" interactions of fermion with vector field (given by the standard replacement $\partial^{\mu} \rightarrow \partial^{\mu} + ieA^{\mu}$) through both of kinetic terms involved in the Lagrangian L (3.3). In this connection, there might appear the question of whether the "fermionic" and "bosonic" type couplings of the ψ field with the vector field A^{μ} have the same coupling constant e. If so, the total Lagrangian with the above "minimal" interaction included, while being non-renormalizible, will still be left gauge invariant. However, generally, these coupling constants are different, which means that the Lagrangian is no more gauge invariant as soon as one takes into account the small "bosonic" type kinetic term in (3.3) being suppressed by the scale M. This is just a type of gauge non-invariance that underlies our model leading eventually to physical Lorentz violation. So, the initially Lorentz invariant theory for fermion-vector field interactions, which possesses a slightly broken gauge invariance, is given by the general Lagrangian[2]

$$L\left(A, \psi\right) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}[i\gamma_{\mu}D^{\mu} - m_0]\psi + \frac{1}{\mathcal{M}}D_{\mu}^{\prime *}\overline{\psi} \cdot D^{\prime\mu}\psi \tag{3.5}$$

containing, apart from the "true fermionic" terms with covariant derivative $D^{\mu} = \partial^{\mu} + ieA^{\mu}$, the "bosonic" type terms as well with $D^{\prime\mu} = \partial^{\mu} + ie^{\prime}A^{\mu}$, either taken with independent charges e and $e^{\prime}$, re-

[2] For simlicity we have not included into the Lagrangian $L\left(A, \psi\right)$ the anomalous magnetic moment type coupling $\frac{e^{\prime\prime}}{\mathcal{M}}F^{\mu\nu}\overline{\psi}\sigma_{\mu\nu}\psi$ which is gauge invariant on its own and appears inessential for what follows. Another simplification is that we have omitted an independent "sea-gull" type coupling $\frac{e^{\prime\prime\prime}}{\mathcal{M}}A_{\mu}^{2}\overline{\psi}\psi$ in the Lagrangian (a term like that is already contained in its "bosonic" part), since such a coupling due to the SLIV condition (1) is simply reduced to some inessential correction to the fermion mass term. All things considered, the Lagrangian $L\left(A, \psi\right)$ gives in fact the most general extension of QED in $\frac{1}{\mathcal{M}}$ order, taken under the Lorentz and extra discrete and global symmetry restrictions discussed above.

spectively. Remarkably, despite the fact that both the “fermionic” and “bosonic” parts of the Lagrangian (3.5) are individually gauge invariant, gauge invariance is in fact broken when they are taken together. As a result, though this Lagrangian practically (i.e. neglecting the last term in (3.5)) does not differ from a conventional QED Lagrangian, provided that the vector field A_μ is associated with a photon, a drastic difference appears when this field develops a vev and the SLIV occurs.

Actually, putting the SLIV parameterization (2.3) into our basic Lagrangian (3.5) one comes to the truly Goldstonic model for the QED. This model contains, among other terms, the inappropriately large (while false) Lorentz violating fermion bilinear $-eM\overline{\psi}(n_\mu\gamma^\mu/n^2)\psi$, which appears when the effective Higgs field expansion (as is given in the parametrization (2.3)) in true Goldstone modes a_μ is applied to the fermion current interaction term $-\overline{\psi}\gamma_\mu A^\mu\psi$ in the "fermionic" part of the Lagrangian $L\,(A,\psi)$. However, due to local invariance of this part, this bilinear term can be gauged away by making an appropriate redefinition of the fermion field $\psi\rightarrow e^{-ie\omega(x)}\psi$ with a gauge function $\omega(x)$ linear in coordinates, $\omega(x)=(n_\mu x^\mu/n^2)M$. Meanwhile, the small "bosonic" part being gauge non-invariant is appropriately changed under this redefinition. So, one eventually arrives at the essentially non-linear SLIV Lagrangian for photon-fermion interaction with the significantly modified fermion bilinear terms

$$\begin{aligned}\mathcal{L}\,(a_\mu,\psi) &= L\,\left(A_\mu\rightarrow a_\mu+n_\mu(a^2/2M+\cdots),\psi\right)+ \\ & -i\Delta e\frac{M}{\mathcal{M}}\frac{n_\mu}{n^2}\overline{\psi}\overleftrightarrow{\partial^\mu}\psi+(\Delta e)^2n^2\frac{M^2}{\mathcal{M}}\overline{\psi}\psi\end{aligned} \tag{3.6}$$

where we have explicitly indicated that the vector field A_μ in the starting Lagrangian L (3.5) is replaced by the pure Goldstone field a_μ associated with the photon (appearing in the gauge $n_\mu a^\mu = 0$) plus the effective Higgs field expansion in (2.3). We also retained the notation ψ for the redefined fermion field and denoted, as usually, $\overline{\psi}\overleftrightarrow{\partial^\mu}\psi = \overline{\psi}(\partial^\mu\psi) - (\partial^\mu\overline{\psi})\psi$. Note that the extra fermion bilinear terms[3] given in the second line in (3.6) are produced just due to the gauge invariance breaking that is determined by the electromagnetic charge difference $\Delta e = e' - e$ in the starting Lagrangian L (3.5). As a result, there appears the entirely new, SLIV inspired, dispersion relation for a charged fermion (taken with 4-momentum p_μ) of the type

$$p_\mu^2 \cong [m + 2\delta(p_\mu n^\mu/n^2)]^2, \quad m = m_0 - \delta^2 n^2 \mathcal{M} \tag{3.7}$$

given to an accuracy of $O(m^2/M^2)$. Here δ stands for the small characteristic, positive or negative, parameter $\delta = (\Delta e)M/M$ of the physical Lorentz violation that reflects the joint effect as given, from the one hand, by the SLIV scale M and, from the other, by the charge difference Δe being a measure of an internal gauge non-invariance. Notably, the space-time by itself still possesses Lorentz invariance, however, fermions with the SLIV contributing into their total mass $m = m_0 - \delta^2 n^2 M$ propagate and interact in it in the Lorentz non-covariant way[4]. At the same time, the photon dispersion relation

[3]Notably, from a general SME point of view one could say that just this form of physical Lorentz violation known as "e-term" breaking [2] appears to dominate, due to the genuine SLIV pattern considered, over many other Lorentz breaking terms emerging in the SME.

[4]Note also that the fermion dispersion relation (3.7) is substantially different from the dispersion relations extensively used before [3] where just the mass squared happened to be shifted in the preferred frame rather than the mass by itself as in Eq.(3.7).

is retained in order $1/M$ considered [5].

Let us now try to estimate a possible scale of Lorentz violation and a numerical value of the parameter δ being in essence the only measure of the physical Lorentz violation in our model. Some estimation could follow from the naturalness requirement that the free fermion mass presented in the Goldstonic QED Lagrangian (3.6) and, specifically the mass of the lightest charged fermion which is the electron mass, should not be significantly disturbed by the Lorentz violation. Otherwise possible fine tuning between the SLIV contribution to this mass and its starting value would become necessary. Proposing the SLIV contributed total electron mass m_e to remain of the same order as the starting mass m_{0e}, one comes from (3.7) to the inequality $\delta^2 M \lesssim m_e$. Remarkably, the characteristic parameter δ depends on neither the SLIV scale M nor on the charge difference Δe individually but on their product only, and, for the above "stability condition" against the SLIV contribution to the electron mass, it is generally given by the range of values

$$\delta = (\Delta e)M/\mathcal{M}, \quad |\delta| \lesssim \overline{\delta} \equiv \sqrt{m_e/\mathcal{M}} \tag{3.8}$$

which for a sufficiently high mass scale M happens by itself (as we see below) to be of a certain interest for current high-energy tests of special relativity. Particularly, when taking just the Planck mass M_P for the highest scale in the theory ($M = M_P$), one has the upper

[5] One must, of course, expect that non-gauge invariant photon kinetic terms, changing its dispersion relation, are also generated through radiative corrections. But these terms are down by high orders in $1/\mathcal{M}$ relative to the basic $F_{\mu\nu}^2$ term taken, and, therefore, can be neglected.

limits

$$|\delta| \lesssim \overline{\delta} = 6.5 \times 10^{-12}, \quad M \lesssim 10^{8}(e/\Delta e)\ GeV \tag{3.9}$$

for the δ parameter and the Lorentz violation scale M, respectively.

Before proceeding to applications, let us note that, in the order $1/M$ considered, all other particles apart from charged fermions, such as photon, neutrinos, weak bosons etc. are proposed to satisfy the standard dispersion relations. Inclusion of new charged fermions into the Goldstonic QED Lagrangian (3.6) will in general increase the number of the SLIV parameters in theory by assigning to every fermion species f (being some lepton or quark) its own δ_f parameter . These parameters, as is seen from (3.8), will actually differ from one another by the corresponding charge differences $(\Delta e)_f$ only. This immediately leads to the conclusion that the δ parameters for particles and antiparticles must be equal but of opposite sign. Apart from that, some of the charge differences might appear to be equal if certain symmetries for leptons and quarks are postulated; say, grand unified symmetry inside a lepton-quark family and/or flavor symmetry between families.

3.2 Some immediate applications

One may now see that, due to spontaneous Lorentz violation resulting in the new dispersion relation (3.7) for charged fermions, the kinematics of processes in which such fermions are participating is substantially changed. At low energies these changes can be neglected, but at high energies they may play crucial role. As a result, some of allowed processes appear to be suppressed at high energies and, on the contrary, some of suppressed processes are now allowed to go. This could substantially change the particle phenomenology at high energies that would lead to some new observations, as well as corrections to the early Universe scenario. Certain of these processes were previously discussed in direct Lorentz violation scenarios [17, 18]. Predictions of our SLIV model appear in fact to be more distinctive being dependent on only a few SLIV parameters δ (3.8) assigned to elementary charged fermions, quarks and leptons. Actually, all changes as compared with a conventional QED can readily be derived replacing masses m_f of these fermions by their non-covariant "effective" masses

$$m_f^* \equiv \sqrt{p_\mu^2} \cong |m_f + 2\boldsymbol{\delta}_f p_0| \,, \tag{3.10}$$

as follows from the above dispersion relation (3.7), where we also introduced a modified (two-component) parameter $\boldsymbol{\delta}_f$ which is equal to $\boldsymbol{\delta}_f = \delta_f$ for the time-like SLIV and $\boldsymbol{\delta}_f = \delta_f \cos\theta$ for the space-like SLIV, respectively. Note that in a high-energy region that we are interested in, the scalar product $p_\mu n^\mu/n^2$ in Eq. (3.7) for the space-like SLIV ($n^2 < 0$) with the angle θ between a fermion 3-momentum $\overrightarrow{p}$ and the Lorentz violation direction vector $\overrightarrow{n}$ just reduces to $p_\mu n^\mu/n^2 = |\overrightarrow{p}| \cos\theta \cong p_0 \cos\theta$.

Consideration of composite hadron states, mesons and baryons, in our model needs further clarification. Generally, one could assign to each of these composites its own δ parameter, or its own effective mass m^* (3.10) which would lead to a plethora of new SLIV parameters in the model. However, we propose the following simple rules for composites that might naturally work. Actually, one may treat SLIV features of hadrons solely based on their quark content so that their effective masses are additively combined with those of quarks and antiquarks involved, both taken at the same energy E in a preferred frame. So, for some meson φ consisting of quark q_1 and antiquark $\overline{q}_2$ this effective mass might look like

$$m_{\varphi}^{*} = |m_{\varphi} + 2(\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2)E| \tag{3.11}$$

where we have used that the δ parameter for antiquark has an opposite sign ($\delta_{q_1} \equiv \delta_1$, $\delta_{\overline{q}_2} = -\delta_{q_2} \equiv -\delta_2$), as was indicated at the end of Sec.2, and replaced the sum of the current quark masses $m_1 + m_2$ in (3.11) by the meson invariant mass m_{φ}. This replacement seems to be a quite good approximation for mesons consisting of heavy c, b and t quarks, but not for mesons consisting of light quarks u, d and s, whose current masses $m_{u,d,s}$ hardly provide masses of the corresponding mesons (and baryons). The point is, however, that the color interaction converting these current quark masses into the constituent quark ones (and leading eventually to the physical hadron masses) is presumably Lorentz invariant, so that the non-covariant part in the meson effective mass (3.11) with δ parameters depending solely on the quark electric charge differences $(\Delta e)_{q_{1,2}}$ seems to be basically preserved. Analogously, dispersion relations of baryons are always frame-dependent being determined by a particular quark content in its effective mass

$$m_{B}^{*} = |m_{B} + 2(\boldsymbol{\delta}_1 + \boldsymbol{\delta}_2 + \boldsymbol{\delta}_3)E| \tag{3.12}$$

provided that the baryon B with invariant mass m_B is composed of quarks q_1, q_2 and q_3 with parameters $\delta_{1,2,3}$.

A few simple remarks are in order. As is readily seen from Eq.(3.11), mesons which are diagonal in the quark flavors, such as π^0, η, ρ^0, ϕ, J/Ψ etc., have zero δ parameters and thus they hold standard dispersion relations. Furthermore, mesons and baryons with the same quark content possess equal δ parameters and, therefore, have alike effective masses. And, in a similar manner as in the elementary fermion case, the δ parameters for composite hadrons and their antiparticles appear to be equal but of opposite sign.

3.2.1 GZK cutoff revised

One of the most interesting examples where a departure from Lorentz invariance can essentially affect a physical process is the transition $p + \gamma \to \Delta$ which underlies the Greisen-Zatsepin-Kouzmin (GZK) cutoff for ultra-high energy (UHE) cosmic rays [19]. According to this idea primary high-energy nucleons (p) should suffer an inelastic impact with cosmic background photons (γ) due to the resonant formation of the first pion-nucleon resonance $\Delta(1232)$, so that nucleons with energies above $\sim 5 \cdot 10^{19} eV$ could not reach us from further away than $\sim 50\ Mpc$. During the last decade there were serious indications [20] that the primary cosmic-ray spectrum extends well beyond the GZK cutoff, though presently the situation is somewhat unclear due to a certain criticism of these results and new data that recently appeared [21]. However, no matter how things will develop, we could say that according to the new fermion dispersion relation (3.7) the GZK cutoff will necessarily be changed (increased or decreased depending on the sign of the corresponding δ parameter) at superhigh energies. Remarkably, for the Planck mass scale case in the theory ($M = M_P$) the above transition, providing this cutoff, appears to be significantly weakened (or even completely undone) just around the aforementioned GZK energy region, as one can see from the δ parameter value range (3.9) calculated for this case.

Really, we must replace the fermion masses in a conventional proton threshold energy for this process by their effective masses $m^*_{p,\Delta} \cong |m_{p,\Delta} + 2\boldsymbol{\delta} E_{p,\Delta}|$ which can be taken with equal δ parameters as for composite states having a similar quark content ($\delta \equiv \delta_p = \delta_\Delta = 2\delta_u + \delta_d$). Using then the approximate equality of their energies, $E_\Delta = E_p + \omega \cong E_p$, since the target photon energies ω are vanishingly small (being a thermal distribution with temperature $T = 2.73\ K$, or $kT \equiv \varpi = 2.35 \times 10^{-4} eV$), one comes to the condition determining the proton energy region in which the foregoing transition is kinematically forbidden for a head-on impact

$$E_p > \frac{m_\Delta^2 - m_p^2}{4[\omega - \boldsymbol{\delta}(m_\Delta - m_p)]} = \frac{6.8}{\omega/\varpi - 8.1\ \boldsymbol{\delta}/\overline{\delta}} \times 10^{20}\ eV. \tag{3.13}$$

As one can readily see, the SLIV modification of the proton threshold energy E_p in Eq.(3.13) might naturally relax the GZK cutoff and even permits UHE cosmic-ray nucleons to travel cosmological distances (when $\omega/\varpi \approx 8.1\ \delta/\overline{\delta}$ with $\overline{\delta}$ given in (3.9)) provided that the δ parameter in Eq.(3.13) is taken positive. Conversely, for its negative values the original GZK cutoff tends to a decrease. Most interestingly, there is predicted some marked spatial anisotropy for primary nucleons in the space-like Lorentz violation case ($\delta = \delta \cos\theta$)

which results in an ordinary GZK cutoff for perpendicular (to the SLIV vector $\overrightarrow{n}$) direction, whereas it is lower or higher for other directions.

3.2.2 Stability of high-energy vector and scalar bosons

Another interesting example is provided by decays of vector and scalar bosons into fermions, no matter whether they all are elementary or composite. Usually these processes are possible if a boson mass m is no less than the sum of fermion invariant masses $m_{1,2}$, but now, when fermions and (composite) bosons can have some effective masses given by Eqs.(11,12,13), these decays at high energies may appear to be kinematically suppressed, as can easily be confirmed. Actually, for a particular two-body decay case this process appears to be banned if the inequality $m^{*} < m_1^{*} + m_2^{*}$ for fermion effective masses $m_{1,2}^{*}$ is satisfied for the minimum total energy of decay products with a given total momentum $\overrightarrow{P}$. It follows that all momenta are collinear in the configuration of minimum total energy and fermion momenta are equal to

$$\overrightarrow{p}_{1,2} = \frac{m_{1,2}}{m_1 + m_2}\overrightarrow{P} \tag{3.14}$$

so that at energies

$$E > \frac{1}{2}(\mathfrak{m} - m_1 - m_2)\frac{m_1 + m_2}{\boldsymbol{\delta}_1 m_1 + m_2\boldsymbol{\delta}_2 - \boldsymbol{\delta}(m_1 + m_2)} \tag{3.15}$$

this boson could appear stable.

Applying this result to the weak W boson decays into quarks and leptons ($m = m_W$, $\delta \equiv \delta_W = 0$) [6] and taking the $\delta_{1,2}$ parameters to be of the same order as those that are required for a weakened GZK cut-off version ($\delta_{1,2} \sim \delta_{p,\Delta} \sim 10^{-12}$), we find that stable W bosons appear at the energy region $\sim 10^{23}eV$ that seems to be somewhat problematic to be directly detected. At the same time, the Z and Higgs bosons which are only related to the flavor-diagonal quark and lepton currents do not change their decay rates with energy since, as already noted, the SLIV effects from particles and antiparticles are expected to be cancelled.

However, the special observational interest may cause charged pion stability at high energies against the standard $\pi \to \mu + \nu$ decays. In contrast to the W boson, the composite charged pion has a non-zero SLIV parameter $\delta_\pi = \delta_u - \delta_d$ (expressed in the up and down quark parameters $\delta_{u,d}$, see Eq.(3.11)) and, therefore, the non-covariant effective mass $m_\pi^* = |m_\pi + 2\boldsymbol{\delta}_\pi E_\pi|$. So, properly adjusting a general formula (3.15) for two-body decays ($m = m_\pi$, $\delta = \delta_\pi$; $m_1 = m_\mu$, $\delta_1 = \delta_\mu$; $m_2 = m_\nu = 0$, $\delta_2 = \delta_\nu = 0$) one eventually has for the threshold energy providing pion stability ($m_\pi^* < m_\mu^*$)

$$E_\pi > \frac{1}{2}\frac{m_\pi - m_\mu}{\boldsymbol{\delta}_\mu - \boldsymbol{\delta}_\pi} \tag{3.16}$$

This energy region, when the muon and pion δ parameter values ($\delta_\mu - \delta_\pi > 0$) are taken to be of the same order $\sim 10^{-12}$ as in the foregoing cases, appears to be significantly lower than that for the stable W boson, being just near the GZK cutoff energy $\sim 10^{19}eV$. Thus, the

[6] For the pure leptonic decay $W \to l\overline{\nu}$ the equation (3.15) is maximally simplified, $E_B > (m_W - m_l)/2\boldsymbol{\delta}_l$, since neutrino is presumably massless and has a normal dispersion relation ($m_2 = 0$ and $\delta_2 = 0$).

UHE primary cosmic rays may include stable charged pions that could in principle be detected at current experiments [21], whereas neutral pions being diagonal quark-antiquark composites are left to be very unstable, as they usually are. Again, for the space-like SLIV case the spatial anisotropy is expected according to which the stable charged pions are predicted to be largely located along the SLIV direction $\vec{n}$.

3.2.3 Modified nucleon decays

As a last example we consider an ordinary neutron β decay ($n \to pe^{-}\overline{\nu}$). Since the neutron is heavier than the proton, $m_n > m_p$, usually neutron β decay is allowed, while proton β decay ($p \to ne^{+}\nu$) is kinematically suppressed. However, due to the Lorentz non-invariance their effective masses may grow at high energies in such a way that $m_n^* < m_p^*$ in a preferred frame. This means that neutrons and protons change places - neutrons become stable, whereas protons decay. Using the above general formula (3.15) one can readily find the threshold energy value when this happens,

$$E > \frac{m_n - m_p}{2(\boldsymbol{\delta}_p - \boldsymbol{\delta}_n)} = \frac{m_n - m_p}{2(\boldsymbol{\delta}_u - \boldsymbol{\delta}_d)} \;, \tag{3.17}$$

where we have treated both beta processes as essentially two-body decays with lepton masses ignored. Again with the $\delta_{p,n}$ parameters $\sim 10^{-12}$ taken as in the foregoing examples, one finds to the energy region $E > 10^{18} eV$ which is an active research area for current cosmic-ray experiments [14,15]. At these energies stable neutrons, as follows, can be contained in primary UHE cosmic rays, whereas unstable protons cannot.

To conclude, we have considered some basic applications of the model which happen to be described in terms of a few δ parameters assigned to elementary fermions, quarks and leptons. Our Lorentz violating predictions appear to be quite certain for the above processes being conditioned just by the vector field model of the SLIV. At the same time this minimal model predicts the strictly vanishing effects in many processes (where generally some Lorentz violation might in principle be expected), such as the Z and Higgs boson and photon decays, decays of diagonal quark-antiquark composites (π^0, η, ρ^0, ϕ, J/Ψ etc.), neutrino oscillations and others which have been previously discussed on pure hypothetical grounds [17, 18]

Chapter 4

SM with partial gauge invariance

As it is shown in previous chapter extended QED can comply with all criteria theory is in need, but main problem here is that symmetry of the nature is much larger then U(1) and it is standard model(SM) symmetry which provides realistic description, which was the aim from the beginning. So this conjecture must be applied to SM. While in extended QED we could construct dimension 5 operator $(1/M)\partial_\mu\overline{\psi}\partial^\mu\psi$, it is impossible in case of SM[1] since it is chiral theory and besides there are more then one internal symmetries. Obviously another approach is needed. So, in case of SM we propose a partial rather than an exact hypercharge gauge invariance according to which, while the electroweak theory is basically $SU(2)\otimes U(1)_Y$ gauge invariant being constructed from ordinary covari-

[1]principally it is not hard and can be easily constructed coupling together SM higgs doublet left fermion doublet and right fermion singlet state with derivative and U(1) gauge field, all this symmetrized and suppressed by proper mass scale, but after doing this immediately appears that such operator will be heavily suppressed even after symmetry breaking because it will be strengthening only by SM scale in contrast to extended QED case where we have instead Lorentz violation scale.

ant derivatives of all fields involved, the $U(1)_Y$ hypercharge gauge field B_μ field is allowed to form all possible polynomial couplings on its own and with other fields invariants. So, the new terms conditioned by the partial gauge invariance in SM may generally have a form

$$U(B) + B_\mu J^\mu(f,h) + B_\mu B_\nu \Theta^{\mu\nu}(f,h,g) + \cdots \tag{4.1}$$

where $U(B)$ contains all possible B field potential energy terms, the second term in (4.1) consists of all hypercharge current-like couplings with SM matter fields (fermions f and Higgs field h), the third term concerns with the possible tensor-like couplings with all SM fields involved (including gauge fields g) and so on. Thus, these new terms (with all kinds of the $SU(3)_c \otimes SU(2) \otimes U(1)_Y$ gauge invariant tensors J^μ, $\Theta^{\mu\nu}$ etc.) "feel" only B field gauge transformations, while remaining invariant under gauge transformations of all other fields. Ultimately, just their "sensitivity" to the former leads to a spontaneous Lorentz violation in SM. Indeed, the vector field SLIV pattern (3.2) by itself can be treated as some gauge transformation with gauge function linear in coordinates, $\omega(x) = (n_\mu x^\mu)M$, and therefore, this violation may only emerge through the gauge non-invariant terms like those in (4.1).

As we show later, even the simplest extension of SM only by the B field potential energy $U(B)$, like that we had in (3.1) for QED, unavoidably leads to SLIV with VEV being developed on some B field component, while its other components convert into the massless Nambu-Goldstone modes. After the standard electroweak symmetry breaking they mix with a neutral W^3 boson of $SU(2)$ leading, as usual, to the massless photon and massive Z boson. The point is, however, that all physical SLIV effects in this minimal case are turned out to be practically insignificant unless one considers some special SLIV interplay with gravity [14, 22] or a possible generation of the SLIV topological defects in the very early universe [23]. Actually, as in the pure SLIV QED [24], one has an ordinary Lorentz invariant low energy physics in an effective SM theory framework with Lorentz breaking effects which may only arise from radiative corrections. The latter is essentially determined by the superheavy (with the SLIV scale order mass) Higgs component contributions and, therefore, is generally expected to be negligibly small at lower energies.

For more clearness and simplicity, one could completely exclude this Higgs component in the theory going to the nonlinear σ-model type SLIV for B field. This procedure, as applied to the QED case [25] (see also [8] and references therein), leads to a directly imposed vector field constraint $A_\mu^2 = n^2 M^2$ which appears virtually in the limit $\lambda \to \infty$ from the potential (3.1), just as it takes place in the original nonlinear σ-model [26] for pions[2]. This constraint provides in fact the genuine Goldstonic nature of QED, as could easily be seen from an appropriate A_μ field parametrization,

$$A_\mu = a_\mu + \frac{n_\mu}{n^2}(M^2 - n^2 a_\nu^2)^{\frac{1}{2}}, \quad n_\mu a^\mu = 0 \tag{4.2}$$

where the pure Goldstone modes a_μ are associated with photon,

[2]This correspondence with the non-linear σ model for pions may be somewhat suggestive, in view of the fact that pions are the only presently known Goldstones and their theory, chiral dynamics [26], is given by the non-linearly realized chiral $SU(2) \times SU(2)$ symmetry rather than by an ordinary linear σ model.

while an effective Higgs mode, or the A_μ field component in the vacuum direction, is given by the square root in (4.2). Indeed, both of these models, linear and nonlinear, are equivalent in the infrared energy domain, where the Higgs mode is considered infinitely massive. For all practical purposes they amount in this limit to QED taken either in the nonlinear gauge $A_\mu^2 = n^2M^2$ for the Lorentz-invariant phase or axial gauge $n_\mu a^\mu = 0$ for the Lorentz-broken one, as was shown in tree [25] and one-loop [8] approximations. We consider for what follows just this nonlinear SLIV alternative in SM and show that the nonlinear SM (or NSM, as we call it hereafter), likewise the nonlinear QED, is observationally equivalent to the conventional SM theory.

So, one way or another, though the photon in QED or the $U(1)_Y$ hypercharge gauge field in SM could very likely be the NG boson, the most fundamental question of whether an actual physical Lorentz violation takes place (that only might point toward such a possibility) is still an open question. Such a violation will necessarily appear, as we see later, if according to the partial gauge invariance conjecture given above, SM is further extended so as to include B field polynomial couplings with other fields invariants as well. The simplest couplings of this kind would be those with dimensionless coupling constants. In the SM framework, they could be given in fact by B field couplings with conventional hypercharge currents of all matter fields involved that was given above by the second term in (4.1). It is clear, however, that their inclusion into the SM Lagrangian would only redefine the hypercharge gauge coupling constant g' which is in essence a free parameter in SM. This means that for the minimal theory with dimensionless coupling constants the partial gauge invariance is basically indistinguishable from an ordinary gauge invariance provided that one completely ignores the superheavy SLIV Higgs component contributions, as is in the NSM framework. However, a crucial difference unavoidably appears when one goes beyond the minimal theory to include as well the tensor-like couplings in (4.1).

Remarkably, the lowest-order couplings in the SM framework, which are in conformity with our partial gauge invariance conjecture and also compatible with all accompanying global and discrete symmetries, appear to be the dimension-6 operators of the type

$$(1/M_P^2)B_\mu B_\nu T^{\mu\nu}(f,g,h) \tag{4.3}$$

where $T^{\mu\nu}$ stands for a sum of the energy-momentum tensor-like bilinears of all SM fields involved, while the Planck mass M_P is taken as the proper inverse scale to these couplings that might be caused by quantum gravity at extra-small distances. As a result, the physical Lorentz violation, in a form that follows from the partial gauge invariance, appears to be naturally suppressed thus being in a reasonable compliance with current experimental bounds. Nonetheless, as we show later in section 4, the couplings (4.3) may lead to a new class of phenomena which could still be of a distinctive observational interest in high energy physics and astrophysics.

The paper is accordingly organized. In section 2 we present a general SLIV model - the simple nonlinear SM (NSM) with the constrained hypercharge gauge field, which is then extended by some other partially gauge invariant high-dimension terms proposedly induced by gravity. In the next section 3 the pure NSM is considered in detail (with all SLIV induced gauge, Yukawa and Higgs interactions) and its observational equivalence to the conventional SM is explicitly demonstrated. When expressed in terms of the pure Goldstone modes, this theory looks essentially nonlinear and contains a variety of Lorentz and CPT violating couplings. Nonetheless, all SLIV effects turn out to be strictly canceled in all lowest order processes some of which are considered in detail. In section 4 we will mainly be focused on the extended nonlinear SM (ENSM) with high-dimension couplings included and consider in detail some immediate physical applications involved. In contrast to previous pure phenomenological considerations, our semi-theoretical approach allows to generally make the more definite predictions (or verify earlier assumptions made ad hoc), and also discuss not only the time-like Lorentz violation but also the space-like case on which the current observational limitations appear to be much weaker. And, finally, in section 5 we conclude.

4.1 The model - a general view

Our starting point is the Standard Model Lagrangian L_{NSM} with nonlinear constraint put on the Abelian $U(1)_Y$ hypercharge gauge field B_μ

$$\begin{aligned} B^2 &= n^2M^2 \qquad (B^2 \equiv B_\mu B^\mu,\ \ n^2 \equiv n_\mu n^\mu,\ \ 1/n^2 = n^2) \\ B_\mu &= b_\mu + \frac{n_\mu}{n^2}(M^2 - n^2 b_\nu^2)^{\frac{1}{2}},\ \ n_\mu b^\mu = 0 \end{aligned} \tag{4.4}$$

with Goldstonic field variables b_μ appeared, just like as we had it for the electromagnetic vector-potential A_μ (4.2). Here again n_μ is a properly oriented unit Lorentz vector, $n^2 = \pm 1$, while M is a proposed SLIV scale.

This nonlinear SM (or NSM) is supposed to be further extended to include some extra partially gauge invariant terms leading eventually to Extended Nonlinear Standard Model (or ENSM). Such extension implies, according to the partial gauge invariance conjecture, an inclusion of all possible B field couplings with other fields invariants in (4.1). This is expected to lead to factual evidence for the physical Lorentz violation at lower energies. Remarkably, the lowest order non-trivial ENSM which is conformity with the chiral nature of SM and all accompanying global and discrete symmetries, is turned out to include the dimension-6 couplings of the type

$$\mathcal{L}_{ENSM} = \mathcal{L}_{NSM} + \frac{B_\mu B_\nu}{M_P^2}(\alpha_f T_f^{\mu\nu} + \alpha_g T_g^{\mu\nu} + \alpha_h T_h^{\mu\nu}) \tag{4.5}$$

describing at the Planck scale M_P the extra interactions of the hypercharge gauge field B_μ (or better to say, its Goldstonic counterpart b_μ) with the energy-momentum tensor like bilinears $T_{f,g,h}^{\mu\nu}$ of all basic fields involved - the matter fermions, and gauge and Higgs bosons, respectively[3]. These tensors are proposed to be all sym-

[3] Note that in the pure QED with vectorlike (rather than chiral) fermions the dimension-5 coupling

metrical and $SU(3)_c \otimes SU(2) \otimes U(1)_Y$ gauge invariant according to our basic conjecture (4.1). So, the physical Lorentz violation, in a form that follows from the partial gauge invariance, appears to be naturally suppressed thus being in a reasonable compliance with current experimental bounds. Nonetheless, as we show in section 4, the extra couplings in (4.5) may lead, basically through the "deformed" dispersion relations of all matter and gauge fields involved, to a new class of phenomena essentially manifesting themselves at ultra-high energies.

It is conceivable, on the other hand, that such extra interaction terms in the L_{ENSM} might arise as a remnant of some operator expansion of the metric tensor $g_{\mu\nu}(x)$ into all possible tensor-valued covariants constructed in quantum gravity. As a result, in the SLIV case, when some of these fields develop VEVs, this could significantly modify the conventional SM interactions at small distances presumably controlled by quantum gravity. Due to the universality of gravity, one could a priori expect an equality of the above interaction constants α_f, α_g and α_h in (4.5)

$$\alpha_f = \alpha_g = \alpha_h = \cdots = \alpha \tag{4.6}$$

for all kinds of matter regardless their properties under SM, while the Lagrangian containing part $(-\eta^{\mu\nu} L_{NSM})$ in the total energy-momentum tensor $T^{\mu\nu}(f, g, h)$ is unessential since it only leads to a proper redefinition of all fields involved. Actually, the contraction

of the type $(1/M_P) A_\mu \overline{\psi} \overleftrightarrow{\partial^\mu} \psi$ satisfying our partial gauge invariance conjecture could also appear [34]. However, for the coventional SM the minimal couplings are proved to be just the terms presented in the ENSM Lagrangian (4.5).

of this part with the shifted hypercharge gauge field B_μ in (4.4) gives in the lowest order the universal factor

$$1-\alpha\frac{M^2n^2}{M_P^2} \tag{4.7}$$

to the whole SM Lagrangian L_{NSM} considered. So, we will consider only "the Lagrangian subtracted" energy-momentum tensor $T^{\mu\nu}(f,g,h)$ in what follows.

The point is, however, that these constants α_f, α_g and α_h in (4.5), even if one starts with an universal gravity-induced constant α at the Planck scale M_P, may appear rather different being appropriately renormalized when running down to lower energies. Supposing some grand unification theory (GUT) for quarks and leptons at a scale being close to the Planck scale one could only deal at start with the above three constants - one universal coupling for the fermion matter (α_f), another coupling for all gauge bosons (α_g) and the third one for Higgs bosons (α_h). Indeed, each of them become different for the fermion, gauge and Higgs submultiplets in GUT, respectively, when going down to the SM energies. Nonetheless, their values at these energies could be in principle calculated from the corresponding radiative corrections. For one example, one could admit that quarks and leptons are joined in the same GUT multiplet, as it appears in the $SO(10)$ model for each of quark-lepton families filling the spinorial 16-plet and, therefore, they have the equal α-coupling (α_f) in this limit. However, due to the radiative corrections this coupling may split into two couplings - one for quarks (α_q) and another for leptons (α_l), respectively. Apart from that, there appear two more coupling constants, namely, those for the left-handed quarks and leptons $(\alpha_{q_L}, \alpha_{l_L})$ and right-handed ones $(\alpha_{q_R}, \alpha_{l_R})$ thus giving in total four different fermion parameters for one quark-lepton family. We will take into account some difference between α-couplings of quarks and leptons but will ignore such a difference for left-handed and right-handed fermions of the same species. Indeed, the corresponding radiative corrections, which basically appear due to C and P non-invariant weak interactions in SM, are expected to be relatively small. So, there are practically left three couplings constants α_f, α_g and α_h in the model (4.5) inside of the quark-lepton family. However, the different quark-lepton families may still have rather different α-couplings that could eventually lead to the flavor-changing processes in our model (see below)[4].

4.2 Nonlinear Standard Model

4.2.1 Hypercharge Goldstone vector boson

We primarily consider the NSM case where, for simplicity, we restrict ourselves to the electronic family only

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \; e_R \tag{4.8}$$

that can be then straightforwardly extended to all matter fermions

[4] In this connection, one could further suppose some family-unified GUT to eventually come to the only starting universal coupling constant α_f for all fermions. One good example could be provided by the $SU(8)$ GUT [35] where all three quark-lepton families are located in its 216-multiplet, $216 = (\overline{5}+10, \overline{3}) + \cdots$, written in terms of an appropriate $SU(5)\otimes SU(3)$ decomposition with the intermediate $SU(5)$ GUT and $SU(3)$ family symmetry. As a result, the above mentioned flavor-changing processes could be then controlled by the subsequent spontaneous breaking of this family symmetry.

observed. Let us rewrite the $U(1)_Y$ gauge field B_μ in terms of its Goldstone counterpart b_μ, which is orthogonal to the preferred Lorentz breaking direction n_μ

$$B_\mu = b_\mu + \frac{n_\mu}{n^2} n_\nu B^\nu \simeq b_\mu + \frac{n_\mu}{n^2} M - \frac{b_\nu^2}{2M} n_\mu \ , \quad n_\mu b^\mu = 0 \tag{4.9}$$

so that in the same order $O(b^2/M)$ the hypercharge field stress-tensor $B_{\mu\nu}$ comes to

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = b_{\mu\nu} - \frac{1}{2M} \left(n_\nu \partial_\mu - n_\mu \partial_\nu \right) \left(b_\rho \right)^2 \tag{4.10}$$

We also explicitly introduce the Goldstonic modes b_μ in the hypercharge covariant derivatives for all matter fields involved that eventually leads to the essentially nonlinear theory for all Goldstonic hypercharge interactions in NSM.

This model might seem unacceptable since it contains, among other terms, the inappropriately large (the SLIV scale M order) Lorentz violating fermion and Higgs fields bilinears which appear when the starting B field expansion (4.9) is applied to the corresponding couplings in SM. However, due to partial gauge invariance, according to which all matter fields remain to possess the covariant derivatives, these bilinears can be gauged away by making an appropriate field redefinition according to

$$(L, e_R, H) \longrightarrow (L, e_R, H) \exp(i \frac{Y_{L,R,H}}{2} g' n^2 M (n_\mu x^\mu) \tag{4.11}$$

So, one eventually has in the same order $O(b^2/M)$ in the expansion

(4.9) the Lagrangian

$$\mathcal{L}_{NSM} = \mathcal{L}_{SM}(B_\mu \to b_\mu) + \mathcal{L}_{nSM} \tag{4.12}$$

where the conventional SM part being expressed in terms of the the hypercharge NG vector boson b_μ is presented in $L_{SM}(B_\mu \to b_\mu)$, while its essentially nonlinear couplings are collected in L_{nSM}

$$\begin{aligned} 2M\mathcal{L}_{nSM} &= -(n\partial)b_\mu \partial^\mu (b_\nu^2) + \frac{1}{2}g' b_\nu^2 \overline{L}\gamma^\mu n_\mu L + g' b_\nu^2 \overline{e}_R \gamma^\mu n_\mu e_R \\ &\quad -\frac{i}{2}g' b_\nu^2 \left[H^+(n_\mu \partial^\mu H) - (n_\mu \partial^\mu H^+)H\right] \end{aligned} \tag{4.13}$$

Note that the SLIV chosen "gauge" $n_\mu b^\mu = 0$ for the b-field is imposed everywhere in the Lagrangian L_{NSM}. Moreover, we take the similar axial gauge for W^i bosons of $SU(2)$ so as to have altogether

$$n_\mu W^{i\mu} = 0 \ , \ \ n_\mu b^\mu = 0 \ \ . \tag{4.14}$$

in what follows. As a result, all terms containing contraction of the unit vector n_μ with electroweak boson fields will vanish in the L_{NSM}.

We see later that NSM, despite the presence of particular Lorentz and CPT violating couplings in its essentially nonlinear part (4.13), does not lead by itself to the physical Lorentz violation until the extra partially gauge invariant terms in the ENSM Lagrangian (4.5) start working.

4.2.2 Electroweak symmetry breaking in NSM

At much lower energies than the SLIV scale M a conventional spontaneous breaking of the internal symmetry $SU(2)\times U(1)_Y$ naturally holds in NSM. This appears when the Higgs field H acquires the VEV through its ordinary potential terms

$$U(H)=\mu_H^2 H^+H+(\lambda/2)(H^+H)^2\ ,\ \ \mu_H^2<0 \tag{4.15}$$

in the electroweak Lagrangian. Since, due to the overall axial gauge adopted (4.14) there is no more a gauge freedom[5] in such theories to exclude extra components in the H doublet, one can parametrize it in the following general form

$$H\equiv\frac{1}{\sqrt{2}}\begin{pmatrix}\phi\\(h+V)e^{i\xi/V}\end{pmatrix}\ ,\qquad V=(-\mu_H^2/\lambda)^{1/2} \tag{4.16}$$

The would-be scalar Goldstone bosons, given by the real ξ and complex $\phi(\phi^*)$ fields, mix generally with Z boson and $W(W^*)$ boson components, respectively. To see these mixings one has to write all bilinear terms stemming from the Higgs boson Lagrangian which consists of its covariantized kinetic term $|D_\mu H|^2$ and the potential energy part (4.15). They are

$$(\partial^\mu h)^2/2+\mu_h^2h^2/2+|M_WW_\mu-i\partial_\mu\phi|^2+(M_ZZ_\mu+\partial_\mu\xi)^2/2 \tag{4.17}$$

where we have used, as usual, the expression for Higgs boson mass $\mu_h^2=\lambda\,|\mu_H^2|$, and also the conventional expressions for W and Z bosons

$$(W_\mu,W_\mu^*)=(W_\mu^1\pm iW_\mu^2)/\sqrt{2}\ ,\quad Z_\mu=\cos\theta\ W_\mu^3-\sin\theta\ b_\mu\ ,\quad \tan\theta\equiv g'/g \tag{4.18}$$

[5] This kind of SM with all gauge bosons taken in the axial gauge was earlier studied [11] in an ordinary Lorentz invariant framework. Also, the SLIV condioned axially gauged vector fields in the spontaneously broken massive QED was considered in [8].

(θ stands for Weinberg angle). They acquire the masses, $M_W = gV/2$ and $M_Z = gV/2\cos\theta$, while an orthogonal superposition of W_μ^3 and b_μ fields, corresponding to the electromagnetic field

$$A_\mu = \cos\theta \; b_\mu + \sin\theta \; W_\mu^3 \tag{4.19}$$

remains massless, as usual. Then to separate the states in (4.17) one needs to properly shift the ξ and ϕ modes. Actually, rewriting the mixing terms in (4.17) in the momentum space and diagonalizing them by the substitutions

$$\phi(k) \to \phi\left(k\right) + M_W \frac{k_\nu W^\nu\left(k\right)}{k^2} \;, \quad \xi(k) \to \xi\left(k\right) - iM_Z \frac{k_\nu Z^\nu\left(k\right)}{k^2} \tag{4.20}$$

one has some transversal bilinear forms for W and Z bosons and the new $\phi(k)$ and $\xi(k)$ states

$$\begin{aligned} &\left| -k_\mu \phi(k) + M_W \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) W^\nu(k) \right|^2 + \\ &+\frac{1}{2}\left[-ik_\mu \xi(k) + M_Z \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) Z^\nu(k) \right]^2 \end{aligned} \tag{4.21}$$

to be separated. As a result, the NSM Lagrangian with the gauge fixing conditions (4.14) included determine eventually the propagators for the massless photon and massive W and Z bosons in the form

$$\begin{aligned} D_{\mu\nu}^{(\gamma)}(k) &= \frac{-i}{k^2 + i\epsilon} \left(g_{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{(nk)} + \frac{n^2 k_\mu k_\nu}{(nk)^2} \right) \;, \\ D_{\mu\nu}^{(W,Z)}(k) &= \frac{-i}{k^2 - M_{W,Z}^2 + i\epsilon} \left(g_{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{(nk)} + \frac{n^2 k_\mu k_\nu}{(nk)^2} \right) \;. \end{aligned} \tag{4.22}$$

(where (nk) stands, as usual, for a contraction $n_\mu k^\mu$). Meanwhile, propagators for the massless scalar fields ϕ and ξ amount to

$$D^{(\phi)}(k) = \frac{i}{k^2}\ , \quad D^{(\xi)}(k) = \frac{i}{k^2} \tag{4.23}$$

These fields correspond to unphysical particles in a sense that they could not appear as incoming or outgoing lines in Feynman graphs, though having some virtual interactions with Higgs boson h, and W and Z bosons that should be taken into account when considering processes with them (see below).

Apart from the bilinear terms (4.17), some new field bilinears appear from the n-oriented Higgs field covariant derivative term $|n^\lambda D_\lambda H|^2$ (see below Eq. (4.45)) when the this NSM is further extended to the ENSM (4.5). They amount to

$$\delta_h[(n_\mu\partial^\mu h)^2 + |n_\mu\partial^\mu\phi|^2 + (n_\mu\partial^\mu\xi)^2] \tag{4.24}$$

where $\delta_h = \alpha_h(M^2/M_P^2)$. Inclusion of the last two terms in the procedure of the $\phi - W$ and $\xi - Z$ separation discussed above, will change a little the form of their propagators (4.22, 4.23) that we do not consider here.

4.2.3 SLIV interactions in NSM

The gauge interactions

The new Goldstonic b-field interactions are given by the Lagrangian L_{NSM} (4.12) and particularly by its pure nonlinear part L_{nSM} (4.13) which includes in the leading order in the inverse SLIV scale $1/M$ the three-linear self-interaction term of the hypercharge vector field $b_\mu = \cos\theta\ A_\mu - \sin\theta\ Z_\mu$ and, besides, the four-linear couplings of this field with left-handed and right-handed fermions, and Higgs boson. All of them have Lorentz noncovariant (preferably oriented) form and, furthermore, they violate CPT invariance as well. For the Higgs part one has in the leading order (again up to b^2/M term in B field expansion (4.9)) using the parametrization (4.16)

$$\mathcal{L}_{nSM}(H) = \frac{1}{2M} g' \left(b_\rho\right)^2 \left[\left(h+V\right)\left(n_\mu \partial^\mu\right)\xi - \frac{i}{2}\left[\phi^*(n_\mu\partial^\mu)\phi - \phi(n_\mu\partial^\mu)\phi^*\right]\right] \tag{4.25}$$

so that the quadrilinar interactions of b_μ field with the would-be Goldstone bosons ξ and $\phi(\phi^*)$ unavoidably emerge. For the properly separated $\phi - W$ and $\xi - Z$ states, which is reached by the replacements (4.20), there appear three- and four-point couplings between all particles involved in the Higgs sector (photon, W, Z, Higgs bosons, and ϕ and ξ fields) as directly follows from the Lagrangian (4.25) taken in the momentum space after corresponding substitutions of (4.20) and $b_\mu = \cos\theta\ A_\mu - \sin\theta\ Z_\mu$, respectively.

Yukawa sector

Now let us turn to the Yukava sector whose Lagrangian is

$$\begin{aligned} \mathcal{L}_{Yuk} &= -G\left[\overline{L}He_R + \overline{e}_R H^+ L\right] = \\ &= -\frac{G}{\sqrt{2}}\left[\left(h+V\right)\overline{e}e + i\xi\overline{e}\gamma^5 e + \overline{e}_R\Phi^*\nu_l + \overline{\nu}_l\Phi e_R\right] \end{aligned} \tag{4.26}$$

Due to the ξ field shift (4.20) one extra Yukava type coupling appears for Z boson, which in the momentum space has the form

$$\mathcal{L}_{Yuk}(Zee) = -\frac{G}{\sqrt{2}} M_Z \frac{k_\nu Z^\nu(k)}{k^2} \bar{e}\gamma^5 e = -\frac{g}{2\cos\theta} m_e \frac{k_\nu Z^\nu(k)}{k^2} \bar{e}\gamma^5 e \tag{4.27}$$

The similar extra coupling appears for the charged W boson as well when it is separated from the ϕ field due to the replacement (4.20).

4.2.4 Lorentz preserving SLIV processes

We show now by a direct calculation of some tree level amplitudes that the spontaneous Lorentz violation, being superficial in the massless nonlinear QED [25, 8] is still left intact in the nonlinear SM. Though when expressed in terms of the pure Goldstone modes, the NSM Lagrangian (4.12, 4.13) looks essentially nonlinear and contains a variety of Lorentz and CPT violating couplings, all SLIV effects turn out to be strictly canceled in all lowest order processes. Specifically, we will calculate matrix elements of two SLIV processes naturally emerging in NSM. One of them is the elastic photon-electron scattering and another is the elastic Z boson scattering on an electron.

Photon-electron scattering

This process in lowest order is concerned with four diagrams one of which is given by the direct contact photon-photon-fermion-fermion vertex generated by the b^2-fermion-fermion coupling in (4.13), while three others are pole diagrams where the scattered photon and fermion exchange a virtual photon, Z boson and ξ field, respectively. Their vertices are given, apart from the standard gauge boson-fermion couplings in $L_{SM}(B_\mu \to b_\mu)$ (4.12), by the SLIV b^3 and b^2-fermion couplings in (4.13) and by the b^2-ξ coupling in (4.25), and also by Yukava couplings (4.26, 4.27).

So, one has first directly from the b^2-fermion coupling the matrix element corresponding to the contact diagram

$$\mathcal{M}_c = i\frac{3g}{4M}\sin\theta\cos\theta(\epsilon_1\epsilon_2)\overline{u}_2\gamma^\rho n_\rho(1+\frac{\gamma^5}{3})u_1 \tag{4.28}$$

when expressing it through the weak isotopic constant g and Weinberg angle θ (where $(\epsilon_1\epsilon_2)$ stands for a salar product of photon polarization vectors $\epsilon_{1\mu}$ and $\epsilon_{2\mu}$).

Using then the vertex for the SM photon-electron coupling,

$$-g\sin\theta\gamma^\mu \tag{4.29}$$

together with vertex corresponding to the SLIV three-photon coupling,

$$-\frac{i}{M}\cos^3\theta\,[(nq)q_\nu g_{\lambda\rho} + (nk_1)k_{1\lambda}g_{\nu\rho} + (nk_2)k_{2\rho}g_{\nu\lambda}] \tag{4.30}$$

(where $k_{1,2}$ are ingoing and outgoing photon 4-momenta and $q = k_2 - k_1$, while $(nk_{1,2})$ and (nq) are their contractions with the unit vector n) and photon propagator (4.22), one comes to the matrix element for the first pole diagram with the photon exchange

$M_{p1} = -i\frac{g}{M}\cos^3\theta\sin\theta(\epsilon_1\epsilon_2)\overline{u}_2\gamma^\mu n_\mu u_1$

Analogously, combining the joint vertex for the Lorentz invariant Z boson-fermion couplings which include both an ordinary SM coupling and extra Yukava coupling (4.27) appearing due to a general parametrization (4.16),

$$i\frac{g}{2\cos\theta}\left[\frac{1}{2}\gamma^{\mu}\left(3\sin^2\theta-\cos^2\theta+\gamma^5\right)-m_e\gamma^5\frac{q_\mu}{q^2}\right] \tag{4.31}$$

with the vertex for the SLIV photon-photon-Z boson coupling,

$$i\frac{\cos^2\theta\sin\theta}{M}\left[(1-\frac{M_Z^2}{q^2})(nq)q_\nu g_{\lambda\rho}+(nk_1)k_{1\lambda}g_{\nu\rho}+(nk_2)k_{2\rho}g_{\nu\lambda}\right] , \tag{4.32}$$

one finds the matrix element corresponding to the second pole diagram with the Z-boson exchange

$$\mathcal{M}_{p2}=-i\ \frac{g}{2M}\sin\theta\cos\theta(\epsilon_1\epsilon_2)\overline{u}_2[\gamma^\mu n_\mu(1-2\cos 2\theta+\gamma^5)/2+\gamma^5(nq)m_e/q^2]u_1 \tag{4.33}$$

where was also properly used Dirac equation for on-shell fermions and Z-boson propagator (4.22).

And lastly, the third pole diagram with the ξ field exchange include two vertices, the first corresponds to Yukava ξee coupling (4.26),

$$\frac{g}{2\cos\theta}\frac{m_e}{M_Z}\gamma^5 \tag{4.34}$$

while the second to the SLIV ξ-photon-photon one (4.25)

$$M_Z\cos^2\theta\sin\theta(\epsilon_1\epsilon_2)(nq) \tag{4.35}$$

that leads, using the ξ field propagator (4.23), to the matrix element

$$\mathcal{M}_{p3}=i\frac{g}{2M}\frac{m_e}{q^2}\sin\theta\cos\theta(\epsilon_1\epsilon_2)(nq)\overline{u}_2\gamma^5u_1 \tag{4.36}$$

Putting together all these contributions one can readily see that the total SLIV induced matrix element for the Compton scattering taken in the lowest order precisely vanishes,

$$\mathcal{M}_{SLIV}(\gamma + e \to \gamma + e) = \mathcal{M}_c + \mathcal{M}_{p1} + \mathcal{M}_{p2} + \mathcal{M}_{p3} = 0 \ . \tag{4.37}$$

Z boson scattering on electron

For this process there are similar four diagrams - one is the Z-Z-fermion-fermion contact diagram and three others are pole diagrams where the scattered Z boson and fermion exchange a virtual photon, Z boson and ξ field, respectively. Their vertices are also given by the corresponding couplings in the nonlinear SM Lagrangian terms (4.12, 4.13, 4.25, 4.26, 4.27). One can readily find that the matrix elements for the contact and pole diagrams differ from the similar diagrams in the photon scattering case only by the Weinberg angle factor

$$\mathcal{M}_c' = \tan^2\theta \mathcal{M}_c \ , \quad \mathcal{M}_{pi}' = \tan^2\theta \mathcal{M}_{pi} \ \ (i = 1, 2, 3) \tag{4.38}$$

so that we have the vanished total matrix element in this case as well

$$\mathcal{M}_{SLIV}(Z + e \to Z + e) = \mathcal{M}_c' + \mathcal{M}_{p1}' + \mathcal{M}_{p2}' + \mathcal{M}_{p3}' = 0 \ . \tag{4.39}$$

Other processes

In the next order $1/M^2$ some new SLIV processes, such as photon-photon, Z-Z, photon-Z boson scatterings, also appear in the tree approximation. Their amplitudes are related, as in the above, to photon, Z boson and ξ field exchange diagrams and the contact b^4 interaction diagrams following from the higher terms in $\frac{b_\nu^2}{M^2}$ in the Lagrangian (4.13). Again, all these four diagrams are exactly cancelled giving no the physical Lorentz violating contributions.

Most likely, a similar conclusion can be derived for SLIV loop contributions as well. Actually, as in the massless QED case considered earlier [8], the corresponding one-loop matrix elements in NSM either vanish by themselves or amount to the differences between pairs of the similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of external four-momenta of the particles involved) that in the framework of dimensional regularization leads to their total cancellation. So, NSM seems to be physically indistinguishable from a conventional SM.

4.3 Extended nonlinear standard model

4.3.1 The basic bilinear and three-linear terms

We now proceed to a study of ENSM with extra partially gauge invariant terms presented in the total Lagrangian (4.5). We express them through the hypercharge Goldstonic modes b_μ taking equations (4.9) and (4.10) in the lowest order in b field so that we have

$$\mathcal{L}_{ENSM} = \mathcal{L}_{NSM} + \frac{1}{M_P^2}[b_\mu b_\nu + n^2(n_\mu b_\nu + n_\nu b_\mu)M + n_\mu n_\nu M^2]T^{\mu\nu}(f,g,h) \quad (4.40)$$

with the tensor $T^{\mu\nu}(f,g,h)$ taken as a sum

$$T^{\mu\nu}(f,g,h) = \alpha_f T_f^{\mu\nu} + \alpha_g T_g^{\mu\nu} + \alpha_h T_h^{\mu\nu} \quad (4.41)$$

where the corresponding ("the Lagrangian subtracted") energy-momentum tensors of fermions, gauge and Higgs boson are

$$\begin{aligned}
T_f^{\mu\nu} &= \frac{i}{2}\left[\overline{L}\gamma^{\{\mu}D^{\nu\}}L + \bar{e}_R\gamma^{\{\mu}D^{\nu\}}e_R\right] \quad , \\
T_g^{\mu\nu} &= -B^{\mu\rho}B_\rho^\nu - W^{(i)\mu\rho}W_\rho^{(i)\nu} \;, \\
T_h^{\mu\nu} &= (D^\mu H)^+ D^\nu H + (D^\nu H)^+ D^\mu H
\end{aligned} \tag{4.42}$$

which all are symmetrical and gauge invariant. One can then use that W_μ^i bosons $(i = 1, 2, 3)$, likewise the Goldstonic field b_μ, are also taken in the axial gauge (4.14) due to which one has one noticeable simplification - their preferably oriented covariant derivatives amount to ordinary derivatives

$$n_\mu D^\mu(b, W^i) = n_\mu \partial^\mu \; . \tag{4.43}$$

Eventually, one has for the total Lagrangian (4.5) in the leading order (up to b^2/M term in B field expansion (4.9))

$$\mathcal{L}_{ENSM} = \mathcal{L}_{NSM} + \mathcal{L}_{ENSM2} + \mathcal{L}_{ENSM3} \tag{4.44}$$

where the nonlinear SM Lagrangian L_{NSM} (4.12, 4.13) was discussed above, while for the new terms in the extended Lagrangian L_{ENSM} we have only included the bilinear and three-linear terms in fields involved, L_{ENSM2} and L_{ENSM3}, respectively. Just these terms could determine largest deviations from a conventional SM.

Let us consider first these bilinear terms. One can readily see that they appear from a contraction of the last term $n_\mu n_\nu M^2$ in the square bracket in (4.40) with energy momentum tensors $T^{\mu\nu}_{f,g,h}$. As a result, one finally comes to the bilinear terms collected in

$$\begin{aligned}\mathcal{L}_{ENSM2} &= i\delta_f[\overline{L}\,(\gamma^\mu n_\mu n_\nu \partial^\nu)\,L + \bar{e}_R\,(\gamma^\mu n_\mu n_\nu \partial^\nu)\,e_R] \\ &\quad -\delta_g n_\mu n_\nu (B^{\mu\rho}B^\nu_\rho + W^{(i)\mu\rho}W^{(i)\nu}_\rho) + 2\delta_h\,|n_\nu \partial^\nu H|^2\end{aligned} \tag{4.45}$$

containing the presumably small parameters $\delta_{f,g,h} = \alpha_{f,g,h}M^2/M_P^2$ since the SLIV scale M is generally proposed to be essentially lower than Planck mass M_P. These bilinear terms modify dispersion relations for all fields involved, and lead, in contrast to the nonlinear SM given by Lagrangian L_{NSM} (4.12, 4.13), to the physical Lorentz violation (see below).

Let us turn now to the three-linear Lorentz breaking terms in L_{ENSM}. They emerge from the contraction of the term $n^2(n_\mu b_\nu + n_\nu b_\mu)M$ in the square bracket in (4.40) with the energy-momentum tensors $T^{\mu\nu}_{f,g,h}$. One can see that only contractions with derivative terms in them give the nonzero results so that we have for the corresponding couplings for fermions

$$\mathcal{L}_{ENSM3} = n^2\frac{\delta_f}{M}b_\mu\left[i\overline{L}\,(\gamma^\mu n_\nu \partial^\nu + \gamma^\nu n_\nu \partial^\mu)\,L + i\bar{e}_R\,(\gamma^\mu n_\nu \partial^\nu + \gamma^\nu n_\nu \partial^\mu)\,e_R\right] \ . \tag{4.46}$$

They present in fact the new type of interaction of the hypercharge Goldstone field b_μ with the fermion matter which does not depend on the gauge constant value g' at all. Remarkably, the inclusion of other quark-lepton families into the consideration will necessarily lead to the flavour-changing processes once the related mass matrices of leptons and quarks are diagonalized. The point is, however, that all these coupling in (4.46) are further suppressed by the SLIV scale M and, therefore, may only become significant at superhigh energies being comparable with this scale. In this connection, the flavour-changing processes stemming from the less suppressed bilinear couplings (4.45) appear much more important. We will consider these processes later.

4.3.2 Modified dispersion relations

The bilinear terms collected in the Lagrangian L_{ENSM2} lead, as was mentioned above, to modified dispersion relation for all fields involved.

Fermions

Due to the chiral fermion content in the Standard Model we use for what follows the chiral basis for γ matrices

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} , \quad \sigma^\mu \equiv (1, \sigma^i) , \quad \overline{\sigma}^\mu \equiv (1, -\sigma^i) \tag{4.47}$$

and take the conventional notations for scalar products of 4-momenta p_μ, unit Lorentz vector n_μ and 4-component sigma matrices $\sigma^\mu(\overline{\sigma}^\mu)$, respectively, i.e. $p^2 \equiv p_\mu p^\mu$, $(np) \equiv n_\mu p^\mu$, $\sigma \cdot p \equiv \sigma_\mu p^\mu$ and $\sigma \cdot n \equiv \sigma_\mu n^\mu$. We will discuss below Lorentz violation (in a form conditioned by the partial gauge invariance) in the chiral basis for fermions in some detail.

Neutrino. The Lorentz noncovariant terms for neutrino and electron in L_{ENSM2} has a form

$$i\delta_f \left[\bar{\nu}(\gamma^\rho n_\rho)n^\lambda\partial_\lambda\nu + \bar{e}_L(\gamma^\rho n_\rho)n^\lambda\partial_\lambda e_L + \bar{e}_R(\gamma^\rho n_\rho)n^\lambda\partial_\lambda e_R\right] \tag{4.48}$$

So, the modified Weyl equation for the neutrino spinor $u_\nu(p)$ in the momentum space, when one assumes the standard plane-wave relation

$$\nu(x) = u_\nu(p)\exp(-ip_\mu x^\mu) \quad (p_0 > 0)\ , \tag{4.49}$$

simply comes in the chiral basis for γ matrices (4.47) to

$$[(\overline{\sigma}\cdot p) + \delta_f(\overline{\sigma}\cdot n)(np)]u_\nu(p) = 0 \tag{4.50}$$

In terms of the new 4-momentum

$$p'_\mu = \ p_\mu + \delta_f(np)n_\mu \tag{4.51}$$

it acquires a conventional form

$$(\overline{\sigma}\cdot p')u_\nu(p) = 0 \tag{4.52}$$

So, in terms of the "shifted" 4-momentum p'_μ the neutrino dispersion relation satisfies a standard equation $p'^2 = 0$ that gives

$$p'^2 = p^2 + 2\delta(np)^2 + \delta^2 n^2(np)^2 = 0 \tag{4.53}$$

while the solution for $u_\nu(p')$, as directly follows from (4.52), is

$$u_\nu(p) = \sqrt{\sigma\cdot p'}\xi \tag{4.54}$$

where ξ is some arbitrary 2-component spinor.

Electron. For electron, the picture is a little more complicated. In the same chiral basis one has from the conventional and SLIV induced terms (4.48) the modified Dirac equations for the 2-component left-handed and right-handed spinors describing electron. Indeed, assuming again the standard plane-wave relation

$$e(x) = \begin{pmatrix} u_L(p) \\ u_R(p) \end{pmatrix} \exp(-ip_\mu x^\mu) \ , \ \ p_0 > 0 \tag{4.55}$$

one comes to the equations

$$\begin{aligned} (\overline{\sigma} \cdot p')u_L &= mu_R \\ (\sigma \cdot p')u_R &= mu_L \end{aligned} \tag{4.56}$$

where we have written them in terms of 4-momenta p'

$$p'_\mu = p_\mu + \delta_f (np) n_\mu \tag{4.57}$$

being properly shifted in the preferred spacetime direction. Proceeding with a standard squaring procedure one come to another pair of equations

$$\begin{aligned} (\sigma \cdot p')(\overline{\sigma} \ \cdot p')u_L &= m^2 u_L \\ (\overline{\sigma} \cdot p')(\sigma \cdot p')u_R &= m^2 u_R \end{aligned} \tag{4.58}$$

being separated for the left-handed and right-handed spinors. So, in terms of the "shifted" 4-momentum p'_μ again, the electron dispersion relation satisfies a standard equation $p'^2 = m^2$ that gives

$$p'^2 = p^2 + 2\delta_f(np)^2 + \delta_f^2 n^2 (np)^2 = m^2 \tag{4.59}$$

while the solutions for $u_L(p)$ and $u_R(p)$ spinors in the chiral basis taken are

$$u_L(p) = \sqrt{\sigma \cdot p'}\xi \ , \quad u_R(p') = \sqrt{\overline{\sigma} \cdot p'}\xi \tag{4.60}$$

where ξ is some arbitrary 2-component spinor.

Further, one has to derive the orthonormalization condition for Dirac 4-spinors $u(p) = \binom{u_L(p)}{u_R(p)}$ in the presence of SLIV and also the spin summation condition over all spin states of the physical fermion. Let us propose first the orthonormalization condition for the helicity eigenspinors ξ^s

$$\xi^{s\dagger}\xi^{s'} = \delta^{ss'} \tag{4.61}$$

where index s stands to distinguish the "up" and "down" states. In consequence, one has for the Hermitian conjugated and Dirac conjugated spinors, respectively,

$$u^{s\dagger}(p)u^{s'}(p) = 2[p_0 + \delta_f(np)n_0]\delta^{ss'}, \quad \overline{u}^s u^{s'} = 2m\delta^{ss'} \tag{4.62}$$

Note that, whereas the former is shifted in energy p_0 for a time-like Lorentz violation, the latter appears exactly the same as in the Lorentz invariant theory for both the time-like and space-like SLIV.

Analogously, one has the density matrices for Dirac spinors allowing to sum over the polarization states of a fermion. The simple calculation, after using the unit "density" matrix for the generic ξ^s

spinors

$$\xi^s \xi^{s\dagger} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4.63}$$

(summation in the index s is supposed), finally gives

$$u^s(p)\ \overline{u}^s(p) = \begin{pmatrix} m & \sigma \cdot p' \\ \overline{\sigma} \cdot p' & m \end{pmatrix} = \gamma^\mu [p_\mu + \delta_f (np) n_\mu] + m \tag{4.64}$$

when writing it in terms of the conventional Dirac γ matrices (4.47).

Positron. Consider in conclusion the anti-fermions in the SLIV extended theories. As usual, one identifies them with the negative energy solutions. Their equations in the momentum space appear from the plane-wave expression with the opposite sign in the exponent

$$e(x) = \begin{pmatrix} v_L(p) \\ v_R(p) \end{pmatrix} \exp(i p_\mu x^\mu)\ ,\ \ p_0 > 0 \tag{4.65}$$

and actually follow from the equations (4.56), if one replaces $m \to -m$. As a result, their solution have a form

$$v_L(p) = \sqrt{\sigma \cdot p'} \chi\ ,\quad v_R(p) = -\sqrt{\overline{\sigma} \cdot p'} \chi \tag{4.66}$$

where χ stands for some other spinors which are related to the spinors ξ^s. This relation is given, as usual, by charge conjugation C

$$\chi^s = i\sigma_2 (\xi^s)^* \tag{4.67}$$

where the star means the complex conjugation[6]. This form of χ^s

[6] This conforms with a general definition of the C conjugation for the Dirac spinors as an operation

says that this operation actually interchanges the "up" and "down" spin states given by ξ^s. All other equations for positron states described by the corresponding 4-spinors $v^s(p)$, namely those for the normalization

$$v^{s\dagger}(p)v^{s'}(p) = 2[p_0 + \delta_f(np)n_0]\delta^{ss'}, \quad \overline{v}^s v^{s'} = -2m\delta^{ss'} \tag{4.68}$$

and density matrices

$$u^s(p)\ \overline{u}^s(p) = \begin{pmatrix} -m & \sigma\cdot p' \\ \overline{\sigma}\cdot p' & -m \end{pmatrix} = \gamma^\mu[p_\mu + \delta_f(np)n_\mu] - m\ , \tag{4.69}$$

also straightforwardly emerge. One can notice, that, apart from the standard sign changing before the mass term all these formulas are quite similar to the corresponding expressions in the positive solution case.

Gauge bosons

To establish the form of modified dispersion relations for gauge fields one should take into account, apart from their standard kinetic terms in the NSM Lagrangian (4.12), the quadratic terms appearing from SLIV (4.45).

Photon. Let us consider first the photon case. The modification of photon kinetic term appears from the modifications of kinetic terms for B and W^3 gauge fields both taken in the axial gauge that, due to the invariant quadratic form of the SLIV contribution

$u(p)^c = C\overline{u}(p)^T = i\gamma_2 u^*(p_0, p_i)$, where one identifies $u(p)^c = v(p)$, while C matrix is chosen as $i\gamma_0\gamma_2$.

(4.45), leads to the same modification for the physical fields of Z boson (4.18) and photon (4.19). So, constructing kinetic terms for the photon in the momentum space one readily finds its modified dispersion relation

$$k^2 + 2\delta_g (nk)^2 = 0 \ , \quad \delta_g = \alpha_g (M^2/M_P^2) \tag{4.70}$$

while its SLIV modified propagator has a form

$$D^{\mu\nu} = \frac{-i}{k^2 + 2\delta_g (nk)^2 + i\epsilon} \left[g^{\mu\nu} - \frac{1}{1 + 2\delta_g n^2} \left(\frac{n_\mu k_\nu + k_\mu n_\nu}{(nk)} - n^2 \frac{k_\mu k_\nu}{(nk)^2} + 2\delta_g n_\mu n_\nu \right) \right] \tag{4.71}$$

This satisfies the conditions

$$n^\mu D^{\mu\nu} = 0 \ \ , \qquad k^\mu D^{\mu\nu} = 0 \tag{4.72}$$

where the transversality condition in (4.72) is imposed on the photon "mass shell" which is now determined by the modified dispersion relation (4.70). Clearly, in the Lorentz invariance limit ($\delta_g \to 0$) the propagator (4.71) goes into the standard propagator taken in an axial gauge (4.22).

W and Z bosons. Analogously, constructing the kinetic operators for the massive vector bosons one has the following modified dispersion relations for them

$$k^2 + 2\delta_g (nk)^2 = M_{Z,W}^2 \tag{4.73}$$

To make the simultaneous modification of their propagators, one also should take into account the terms emerged from the Higgs sector. These terms appear when, through the proper diagonalization, the Higgs bilinears decouple from those of the massive W and Z bosons. Due to their excessive length we do not present their modified propagators here.

Higgs boson

For Higgs boson (with 4-momentum k_μ and mass μ_h), we have from the properly modified Klein-Gordon equation, appearing from its basic Lagrangian (4.17) taken together with the last term in L_{ENSM2} (4.45), the dispersion relation

$$k^2 + 2\delta_h(nk)^2 = \mu_h^2 \ , \ \ \delta_h = \alpha_h(M^2/M_P^2) \ . \qquad (4.74)$$

4.3.3 Lorentz breaking SLIV processes

We are ready now to consider the SLIV contributions into some physical processes. They include as ordinary processes where the Lorentz violation gives only some corrections, being quite small at low energies but considerably increasing with energy, so the new processes being entirely determined by SLIV in itself. Note that the most of these processes were considered earlier [17, 18], largely on the pure phenomenological ground. We try here to discuss them in our semi-theoretical ENSM framework that allows us to make sometimes more definite predictions or verify some earlier assumptions made ad hoc. Actually, our model contains only three SLIV parameters δ_f, δ_g and δ_h rather than a variety of phenomenological parameters being introduced individually for each process involved. Indeed, one (or, at most, two) more fermion parameters should be added in our case when different quark-lepton families and related flavor-changing processes are also considered.

Another important side of our consideration is that for every physical process we take into account, together with the direct contributions of the SLIV couplings in the Lagrangian, the Lorentz violating contributions appearing during the integration over phase space. The latter for the most considered processes is still actually absent in the literature. Specifically, for decay processes, we show that when there are identical particles (or particles belonging to the same quark-lepton family) in final states one can directly work with their SLIV shifted 4-momenta (see, for example (4.59)) for which the standard dispersion relations hold and, therefore, standard integration over phase space can be carried out. At the same time for the decaying particles by themselves the special SLIV influenced quantity called the "effective mass" may be introduced. Remarkably, all such decay rates in the leading order in the SLIV δ-parameters are then turned out to be readily expressed in terms of the standard decay rates, apart from that the mass of decaying particle is now replaced by its "effective mass".

Our calculations confirm that there are lots of the potentially sensitive tests of the Lorentz invariance, especially at superhigh energies $E > 10^{18} eV$ that is an active research area for the current cosmic-ray experiments [21]. They include a considerable change in the GZK cutoff for ultra-high energy (UHE) cosmic-ray nucleons, possible stability of high-energy pions and weak bosons and, on the contrary, instability of photons, very significant increase of the radiative muon and kaon decays, and some others. In contrast to previous (pure phenomenological) considerations [17, 18], we also discuss the case of the space-like Lorentz violation on which, due to its spatially anisotropic manifestations, the current observational limitations appear to be much weaker.

Higgs boson decay into fermions

We start with calculation of the Higgs boson decay rate into electron-positron pair. The vertex for such process is given by Yukawa coupling

$$\frac{G}{\sqrt{2}} h\bar{e}e \tag{4.75}$$

with the coupling constant G. Properly squaring the corresponding matrix element with the electron and positron solutions given above (4.55, 4.65) one has

$$\begin{aligned} |\mathcal{M}_{he\bar{e}}|^2 &= \frac{G^2}{2}\left(Tr[(p'_\mu\gamma^\mu)(q'_\nu\gamma^\nu)] - 4m^2\right) \\ &= 2G^2(p'_\mu q'^\mu - m^2) \end{aligned} \tag{4.76}$$

where p'_μ and q'_μ are the SLIV shifted four-momenta of electron and positron, respectively, defined as

$$\begin{aligned} p'_\mu &= p_\mu + \delta(np)n_\mu \ , \quad p'^2_\mu = m^2 \\ q'_\mu &= q_\mu + \delta(nq)n_\mu \ , \quad q'^2_\mu = m^2 \ . \end{aligned} \tag{4.77}$$

We use then the conservation law for the original 4-momenta of

Higgs boson and fermions

$$k_\mu = p_\mu + q_\mu \tag{4.78}$$

since just these "deformed" 4-momenta still determine the space-time evolution of all freely propagating particles involved rather than their SLIV shifted 4-momenta (for which the above conservation law only approximately works). Rewriting this relation as

$$k_\mu + \delta_f(nk)n_\mu = p'_\mu + q'_\mu \tag{4.79}$$

and squaring it one has, using the relations (4.74) and (4.77),

$$p'_\mu q'^\mu = \mu_h^2/2 - (\delta_h - \delta_f)(nk) - m^2 \tag{4.80}$$

that finally gives for the matrix element (4.76)

$$|\mathcal{M}_{he\bar{e}}|^2 = G^2(\boldsymbol{\mu}_h^2 - 4m^2) \tag{4.81}$$

where we have denoted by μ_h^2 the combination

$$\boldsymbol{\mu}_h^2 = \mu_h^2 - 2(\delta_h - \delta_f)(nk)^2\ . \tag{4.82}$$

This can be considered as an "effective" mass square of Higgs boson which goes to the standard value μ_h^2 in the Lorentz invariance limit. One can also introduce the corresponding 4-momentum

$$k'_\mu = k_\mu + \delta_f(nk)n_\mu\ ,\ \ k'^2_\mu = \boldsymbol{\mu}_h^2 \tag{4.83}$$

which differs from the 4-momentum determined due the Higgs boson dispersion relation (4.74).

So, Lorentz violation due to the matrix element is essentially presented in the "effective" mass of the decaying Higgs particle. Let us turn now to the SLIV part stemming from an integration over the phase space of the fermions produced. It is convenient to come from the "deformed" original 4-momenta $(k_\mu,\ p_\mu,\ q_\mu)$ to the shifted ones $(k'_\mu,\ p'_\mu,\ q'_\mu)$ for which fermions have normal dispersion relations given in (4.77). Actually, possible corrections to such momentum replacement are quite negligible[7] as compared to the Lorentz violations stemming from the "effective" mass (4.82) where they are essentially enhanced by the factor $(nk)^2$. Actually, writing the Higgs boson decay rate in the shifted 4-momenta we really come to a standard case, apart from that the Higgs boson mass is now replaced by its "effective" mass (see below). So, for this rate we still have

$$\Gamma_{he\bar{e}} = \frac{G^2(\boldsymbol{\mu}_h^2 - 4m^2)}{32\pi^2 k'_0}\int \frac{d^3p'd^3q'}{p'_0q'_0}\delta^4(k'-p'-q') \tag{4.84}$$

Normally, in a standard Lorentz-invariant case this phase space integral comes to 2π. Now, for the negligible fermion (electron) mass, $\mu_h^2 >> m^2$ (or more exactly $\delta_f k_0^2 >> m^2$) one has, using the corresponding energy-momentum relations of particles involved,

$$\int \frac{d^3p'd^3q'}{p'_0q'_0}\delta^4(k'-p'-q') \simeq 2\pi\frac{k'_0}{\sqrt{\boldsymbol{\mu}_h^2}} \tag{4.85}$$

[7] Actually, there is the following correspondence between the shifted and original momenta when integrating over the phase space: for the delta functions this is $\delta^4(k'-p'-q') = (1+\delta_f)^{-1}\delta^4(k-p-q)$ (for both time-like and space-like SLIV), while for the momentum differentials there are $\frac{d^3p'd^3q'}{k'_0p'_0q'_0} = (1+\delta_f)^{-3}\frac{d^3pd^3q}{k_0p_0q_0}$ (time-like SLIV) and $\frac{d^3p'd^3q'}{k'_0p'_0q'_0} = (1+\delta_f)^2\frac{d^3pd^3q}{k_0p_0q_0}$ (space-like SLIV). So, one can use in a good approximation the shifted momentum variables instead of the original ones.

that for Higgs boson rate eventually gives

$$\Gamma_{he\bar{e}} \simeq \frac{G^2}{16\pi}\sqrt{\boldsymbol{\mu}_h^2} \simeq \Gamma^0_{he\bar{e}}\left[1-(\delta_h-\delta_f)\frac{(nk)^2}{\mu_h^2}\right] \tag{4.86}$$

The superscript "0" in the decay rate Γ here and below belongs to its value in the Lorentz invariance limit. Obviously, the SLIV deviation from this value at high energies depends on a difference of delta parameters. In the time-like SLIV case for energies $k_0 > \mu_h/\left|\delta_h-\delta_f\right|^{1/2}$ this decay channel breaks down, though other channels like as $h \rightarrow 2\gamma$ (or $h \rightarrow 2$ gluons) may still work if the corresponding kinematical bound $\mu_h/\left|\delta_h-\delta_g\right|^{1/2}$ for them is higher. For the space-like SLIV the effective delta parameter becomes dependent on the orientation of momentum of initial particle as well, and if, for example, α is the angle between $\overrightarrow{k}$ and $\overrightarrow{n}$, threshold energy is given by $k_0 > \mu_h/\left|(\delta_h-\delta_f)\cos^2\alpha\right|^{1/2}$. So, the decay rate may acquire a strong spatial anisotropy at ultra-high energies corresponding to standard short-lived Higgs bosons in some directions and, at the same time, to unusually long-lived ones in others.

Weak boson decays

Analogously, one can readily write the Z and W boson decay rates into fermions replacing in standard formulas the Z and W boson masses by their "effective masses" which similar to (4.82) are given by

$$\mathbf{M}^2_{Z,W} \simeq M^2_{Z,W} - 2(\delta_g-\delta_f)(nk)^2 \tag{4.87}$$

Therefore, for the Z boson decay into the neutrino-antineutrino pair one has again the factorized expression in terms of the Lorentz invariant and SLIV contributions

$$\Gamma_{Z\nu\overline{\nu}} \simeq \frac{g^2}{96\pi\cos^2\theta_w}\sqrt{\mathbf{M}_Z^2} \simeq \Gamma^0_{Z\nu\overline{\nu}}\left[1-(\delta_g-\delta_f)\frac{(nk)^2}{M_Z^2}\right] \tag{4.88}$$

For the Z decay into massive fermions (with masses $m << \sqrt{\mathbf{M}_Z^2}$) one has a standard expression though with the "effective" Z boson mass square M_Z^2 inside rather than an ordinary mass square M_Z^2

$$\Gamma_{Zee} = \frac{g^2(1+r)}{96\pi\cos^2\theta_w}\sqrt{\mathbf{M}_Z^2} = \Gamma^0_{Zee}\left[1-(\delta_g-\delta_f)\frac{(nk)^2}{M_Z^2}\right] \tag{4.89}$$

where, for certainty, we have focused on the decay into electron-positron pair and introduced, as usual, the weak angle factor with $r = -4\sin^2\theta_w\cos 2\theta_w$. One can see that in the leading order in δ-parameters the relation between the total decay rates $\Gamma_{Ze\overline{e}}$ and $\Gamma_{Z\nu\overline{\nu}}$ remains the same as in the Lorentz invariant case.

As to the conventional W boson decay into the electron-neutrino pair, one can write in a similar way

$$\Gamma_{We\nu} \simeq \frac{g^2}{48\pi}\sqrt{\mathbf{M}_W^2} \simeq \Gamma^0_{We\nu_e}\left[1-(\delta_g-\delta_f)\frac{(nk)^2}{M_W^2}\right] \tag{4.90}$$

So, again as was in the Higgs boson case, Z boson and W boson at energies $k_0 > M_{Z,W}/\sqrt{\delta_g-\delta_f}$ tend to be stable for the time-like SLIV or decay anisotropically for the space-like one.

Photon decay into electron-positron pair

Whereas the above decays could contain some relatively small SLIV corrections, the possible photon decay, which we now turn to, is entirely determined by the Lorentz violation. Indeed, while physical photon remain massless, its "effective" mass, caused by SLIV, may appear well above of the double electron mass that kinematically allows this process to go.

The basic electromagnetic vertex for fermions in SM is given, as usual

$$-(ie)\,\overline{e}\epsilon_\mu\gamma^\mu e \tag{4.91}$$

where we denoted electric charge by the same letter e as the electron field variable $e(x)$ and introduced the photon polarization vector $\epsilon_\mu(s)$. The fermion dispersion relations in terms of the SLIV shifted four-momenta and the photon "effective" mass have the form (similar to those in the above cases)

$$p'^2_\mu = q'^2_\mu = m^2\ , \qquad \mathbf{M}^2_\gamma \simeq 2\,(\delta_f - \delta_g)\,(n^\mu k_\mu)^2 \equiv\ k'^2_\mu \tag{4.92}$$

Consequently, for the square of the matrix element one has

$$|\mathcal{M}_{\gamma e\overline{e}}|^2 = 4e^2\left[2(p'\epsilon)(q'\epsilon) - \epsilon^2_\mu(m^2 + (p'q'))\right] \tag{4.93}$$

Due to the energy-momentum conservation which generally allows to replace

$$p'_\mu q'_\nu \to \frac{1}{12}\left((\mathbf{M}^2_\gamma - 4m^2)g_{\mu\nu} + 2(\mathbf{M}^2_\gamma + 2m^2)\frac{k'_\mu k'_\nu}{\mathbf{M}^2_\gamma}\right) \tag{4.94}$$

and the summation over the photon polarization states given according to the modified photon propagator (4.71)

$$\epsilon_\mu(s)\epsilon_\nu(s) = -g^{\mu\nu} + \frac{1}{1+2\delta_g n^2}\left(\frac{n_\mu k_\nu + k_\mu n_\nu}{(nk)} - n^2\frac{k_\mu k_\nu}{(nk)^2} + 2\delta_g n_\mu n_\nu\right) \tag{4.95}$$

one finally has for the properly averaged square of the matrix element

$$\left|\overline{\mathcal{M}}_{\gamma e\bar{e}}\right|^2 = \frac{4e^2}{3}(\mathbf{M}_\gamma^2 + 2m^2) \tag{4.96}$$

Therefore, for a calculation of the photon decay rate there is only left an integration over phase space

$$\Gamma_{\gamma e\bar{e}} = \frac{e^2}{24\pi^2 k_0'}(\mathbf{M}_\gamma^2 + 2m^2)\int\frac{d^3p' d^3q'}{p_0' q_0'}\delta^4(k'-p'-q') \tag{4.97}$$

which in a complete analogy with the above Higgs boson decay case (4.84) leads in the limit $M_\gamma^2 >> m^2$ to the especially simple answer

$$\Gamma_{\gamma e\bar{e}} \simeq \frac{e^2}{12\pi}\sqrt{\mathbf{M}_\gamma^2} \simeq \frac{e^2}{12\pi}\sqrt{2\left|\delta\right|}k_0 \tag{4.98}$$

where $\delta = \delta_f - \delta_g$ for the time-like violation and $\delta = (\delta_f - \delta_g)\cos^2\varphi$ for the space-like one with an angle φ between the preferred SLIV direction and the starting photon 3-momentum. Note that, though, as was indicated in [18], the detection of the primary cosmic-ray photons with energies up to $20\ TeV$ sets the stringent limit on the Lorentz violation, this limit belongs in fact to the time-like SLIV case giving $|\delta_f - \delta_g| < 10^{-15}$ rather than to the space-like one which in some directions may appear much more significant.

Radiative muon decay

In contrast, the muon decay process $\mu \to e + \gamma$, though being kinematically allowed, is strictly forbidden in the minimal SM and is left rather small even under some of its known extensions. However, the Lorentz violating interactions in our model may lead to the significant flavor-changing processes both in lepton and quark sector. Particularly, they may raise the radiative muon decay rate up to its experimental upper limit $\Gamma_{\mu e\gamma} < 10^{-11}\Gamma_{\mu e\nu\overline{\nu}}$. The point is that the "effective" mass eigenstates of high-energy fermions do not in general coincide with their ordinary mass eigenstates. So, if we admit that, while inside of the each family all fermions are proposed to have equal SLIV δ-parameters, the different families could have in general the different ones, say, δ_e, δ_μ and δ_τ for the first, second and third family, respectively. As a result, diagonalization of the fermion mass matrices will then cause small non-diagonalities in the energy-dependent part of the fermion bilinears presented in the L_{ENSM2} (4.45), even if initially they are taken diagonal.

Let us consider, as some illustration, the electron-muon system ignoring for the moment possible mixings of electrons and muons with tau leptons. To this end, the Lagrangian L_{ENSM2} is supposed to be extended so as to include the muon bilinears as well. Obviously, the leading diagrams contributing into the $\mu \to e + \gamma$ are in fact two simple tree diagrams where muon emits first photon and then goes to electron due to the "Cabibbo rotated" bilinear couplings (4.45) or, on the contrary, muon goes first to electron and then emits photon. Let us ignore this time the pure kinematical part of the SLIV contribution following from the deformed dispersion relations of all particles involved thus keeping in mind only its "Cabibbo rotated" part in the properly extended bilinear couplings (4.45). In this approximation the radiative muon decay rate is given by

$$\Gamma_{\mu e\gamma} = \frac{e^2}{32\pi}\frac{(pn)^3}{m_\mu^2}(\delta_\mu - \delta_e)^2 \sin^2 2\theta \tag{4.99}$$

where p is the muon 4-momentum and θ is the corresponding mixing angle of electron and muon. Taking for their starting mass matrix m_{ab} the Hermitian matrix with a typical $m_{11} = 0$ texture form [27]

$$m_{ab} = \begin{pmatrix} 0 & b \\ b & c \end{pmatrix} , \tag{4.100}$$

one has

$$\sin^2 2\theta = 4\frac{m_e}{m_\mu} . \tag{4.101}$$

As one can see, though the decay rate (4.99) is in fact negligibly small when muon is at rest, this rate increases with the cube of the muon energy and becomes the dominant decay mode at sufficiently high energies. If we admit that there are still detected the UHE primary cosmic ray muons possessing energies around $10^{19} eV$ [21] the following upper limit for the SLIV parameters stems

$$|\delta_\mu - \delta_e| < 10^{-24} \tag{4.102}$$

provided that the branching ratio $\Gamma_{\mu e\gamma}/\Gamma_{\mu e\nu\overline{\nu}}$ at these energies is taken to be of the order one or so. This suggests, as one can see, a rather sensitive way of observation of a possible Lorentz violation through the search for a lifetime anomaly of muons at ultra-high energies

(UHE).

The GZK cutoff revised

One of the most interesting examples where a departure from Lorentz invariance can essentially affect a physical process is the transition $p + \gamma \to \Delta$ which underlies the Greisen-Zatsepin-Kouzmin (GZK) cutoff for UHE cosmic rays [19]. According to this idea primary high-energy nucleons (p) should suffer an inelastic impact with cosmic background photons (γ) due to the resonant formation of the first pion-nucleon resonance $\Delta(1232)$, so that nucleons with energies above $\sim 5 \cdot 10^{19} eV$ could not reach us from further away than $\sim 50\ Mpc$. During the last decade there were some serious indications [20] that the primary cosmic-ray spectrum extends well beyond the GZK cutoff, though presently the situation is somewhat unclear due to a certain criticism of these results and new data that recently appeared [21]. However, no matter how things will develop, we could say that according to the modified dispersion relations of all particles involved the GZK cutoff will necessarily be changed at superhigh energies.

Actually, one may expect that the modified dispersion relations for quarks will change dispersion relations for composite hadrons (protons, neutrons, pions, Δ resonances etc.) depending on a particular low-energy QCD dynamics appearing in each of these states. In general, one could accept that their dispersion relations have the same form (4.59) as they have for elementary fermions, apart from that their SLIV δ parameters values may differ. So, for the proton and Δ there appear equations,

$$P_{p,\Delta}^2 = m_{p,\Delta}^2 - 2\delta_{p,\Delta}\,(nP_{p,\Delta})^2 = \mathbf{m}_{p,\Delta}^2 \tag{4.103}$$

respectively, which determine their deformed dispersion relations and corresponding "effective" masses (where $P_\mu = (E, P_i)$ stands for the associated 4-momenta). Really, we must replace the fermion masses in a conventional proton threshold energy for the above process

$$E_p \geq \frac{m_\Delta^2 - m_p^2}{4\omega} \tag{4.104}$$

by their "effective" masses $m_{p,\Delta}^2$, where the target photon energies ω are vanishingly small ($\omega \sim 10^{-4} eV$) and, therefore, its SLIV induced "effective mass" can be ignored (that gives an approximate equality of the fermion energies, $E_\Delta = E_p + \omega \cong E_p$). As a result, the modified threshold energy for the UHE proton scattering on the background photon via the intermediate Δ particle production is happened to be

$$E_p \geq \frac{m_\Delta^2 - m_p^2}{2\omega + \sqrt{4\omega^2 + 2(\delta_\Delta - \delta_p)(m_\Delta^2 - m_p^2)}} \tag{4.105}$$

Obviously, if there is time-like Lorentz violation and, besides, $\delta_p - \delta_\Delta > 2\omega^2/(m_\Delta^2 - m_p^2)$ this process, as follows from (4.105) becomes kinematically forbidden at all energies, while for other values of δ parameters one could significantly relax the GZK cutoff. The more interesting picture seems to appear for the space-like SLIV with $\delta_p - \delta_\Delta > 2\omega^2/\left(m_\Delta^2 - m_p^2\right)\cos^2\alpha$, where α is the angle between the initial proton 3-momentum and preferred SLIV direction fixed by the unit vector $\vec{n}$. Actually, one could generally observe different cutoffs for different directions, or not to have them at all for some other directions thus permitting the UHE cosmic-ray nucleons to travel over cosmological distances.

Other hadron processes

Some other hadron processes, like as the pion or nucleon decays, studied phenomenologically earlier [18] are also interesting to be reconsidered in our semi-theoretical framework. Departures from Lorentz invariance can also modify the rates of allowed hadron processes, such as $\pi \to \mu + \nu$ and $\pi \to 2\gamma$. In our model these rates can be readily written replacing the mass of the decaying pion by its "effective masses" being determined independently for each of these cases. So, one has them again in the above mentioned factorized forms (in the leading order in δ-parameters)

$$\begin{aligned} \Gamma_{\pi\mu\nu} &\simeq \Gamma^0_{\pi\mu\nu}\left[1-(\delta_\pi-\delta_f)\frac{(nk)^2}{m_\pi^2}\right] \\ \Gamma_{\pi\gamma\gamma} &\simeq \Gamma^0_{\pi\gamma\gamma}\left[1-3(\delta_\pi-\delta_g)\frac{(nk)^2}{m_\pi^2}\right] \end{aligned} \tag{4.106}$$

where we have used that their standard decay rates are proportional to the first and third power of the pion mass, respectively. Therefore, the charged pions at energies $k_0 > m_\pi/\sqrt{\delta_\pi - \delta_f}$ and neutral pions at energies $k_0 > m_\pi/\sqrt{3(\delta_\pi - \delta_g)}$ may become stable for the time-like SLIV or decay anisotropically for the space-like one. As was indicated in [18], even for extremely small for δ-parameters of the order $10^{-24} \div 10^{-22}$ this phenomenon could appear for the presently studied UHE primary cosmic ray pions possessing energies around $10^{19} eV$ and higher.

As in the lepton sector, there also could be the SLIV induced flavor-changing transitions in the quark sector leading to the flavor-changing processes for hadrons. The SLIV induced radiative quark decay $s \to d+\gamma$ is of a special interest. This could make the radiative hadron decays $K \to \pi + \gamma$ and $\Sigma(\Lambda) \to N + \gamma$ to become dominant at ultra-high energies just like as we had this for the radiative muon decay. Again, an absence of kaons and hyperons at these energies or marked decrease of their lifetime could point to the fact that Lorentz invariance is violated.

Explaining OPERA data

Recently very active and in some sense controversial is discussion about OPERA experiment. Initially it was intended to study neutrino oscillation on the beam of μ neutrinos directed from CERN to Gran Sasso laboratory, but as an addition neutrinos have been observed as a superluminal particles, as the group claims. The neutrinos covered 730 km in 60ns less time then photon would, thus fixing $\delta_{\nu_\mu} = 5 * 10^{-5}$. Also neutrino beam does not show sign of the beam energy depletion. Such a big δ_{ν_μ} within our semi-theoretical ENSM framework can be explained only in space-like violation case, because our framework makes fermions from same generation to have same maximum aberration from speed of light, which would contradict with constraint for muon aberration in time-like violation case.

Generally, neutrino is the stable particle and its decay is kinematically forbidden, but now with such aberration effective mass is energy dependent and for average energy 30 Gev it is

$$m_{ef} = \sqrt{\delta_\nu}E \simeq 2 * 10^{-1}(Gev) = 200(Mev)$$

which already makes kinematically possible some of weak decay channels. Those are:

$$\nu_\mu - \ > \ \nu_\mu + f + \overline{f}$$
$$\nu_\mu - \ > \ \nu_\mu + \gamma$$

Fermion-antifermion radiation can be allowed only for $f = e^-, \nu_e$, because all other candidates are too massive for such energy range, but even for them they should have different from muon neutrino δ parameters in order decay(fermion-antifermion radiation) could take place. In our semi-theoretical framework it is not necessary for the fermions from different generation to have same maximum aberration from speed of light, but here we are constrained from the neutrino oscillation observational fact that flavor depended Lorentz violation is constrained at least by $|\delta_i - \delta_j| < 10^{-19}$ $(i, j$ are a flavor indexes) thus making it totally negligible and therefore making fermion-antifermion radiation kinematically impossible(at least at such a low energies).

While fermions emission is forbidden, neutrino can freely radiate photons in all energy range if $\delta_v - \delta_\gamma > 0$, but this process goes via loop diagram(in the weak interaction sector) and therefore is weak and it requires much larger distance then 700 km for neutrinos beam to experience any significant energy depletion from the photon radiation.

Assuming space-like violation, only which can be applied in this case in order to do not contradict with experimental data for different kind of leptons, we understand that aberration from the speed of light is anisotropic and varies from the zero at ortogonal to preferred direction given by $\vec{n}$ to the maximum value at preferred $\vec{n}$ direction. So, effective aberration $\delta_{ef} = \delta \cos^2(\alpha)$ (where α is an angle between $\vec{n}$ and direction of the motion) doesn't change much for wide range of α and in parallel respecting the idea that Lorentz violation should be small, we can conclude that if OPERA experiment is correct, maximum aberration should be order of $\delta \sim 10^{-4} \div 10^{-5}$. Practically observed value in different directions can be much smaller.

Chapter 5

Conclusions

We have studied the nonlinear σ-model type SLIV phenomenon for the vector field both in the properly extended QED and SM theory and some immediate physical and astrophysical consequences following from them. In this connection, the most important conclusions are in order.

(1) We show that nonlinear constraint put on the vector field $A^2 = n^2M^2$(M is the Lorentz violation scale) spontaneously violates Lorentz symmetry and, as consequence, induces the genuine vector Goldstone boson. At the same time this constraint excludes the supermassive Higgs modes from the theory thus leaving only pure vector Goldstone modes. Remarkably, they stay massless, when the corresponding $U(1)$ internal symmetry is exact, but becomes massive if it is spontaneously violated.

(2) We directly demonstrated that despite the variety of the Lorentz (and CPT) violating interactions in the SLIV induced QED and Standard Model, the physical Lorentz violation does not appear, because in any processes all such effects cancel each other both

in the tree and one-loop approximations. Therefore, these theories are physically indistinguishable from the ordinary ones taken in an axial gauge. In this sense we showed that the conventional QED and SM are in essence the spontaneously broken theories with photon appearing as the Nambu-Goldstone boson.

(3) We constructed an alternative theory of QED with non-exact gauge invariance in which together with a Goldstonic photon the Lorentz violation by itself, while being superficial in gauge invariant theory, becomes physically significant. This leads, through properly deformed dispersion relations appearing for charged fermions, to a new class of phenomena which could be of distinctive observational interest in particle physics and astrophysics. They include a significant change in the Greizen-Zatsepin-Kouzmin (GZK) cutoff for UHE cosmic ray nucleons, stability of high-energy pions and W bosons, modification of nucleon beta decays, and some others.

(4) We propose the extended Standard Model by terms which could follow from quantum gravity at very small distances. Arguing that exact gauge invariance may hide some generic features of SM, that could otherwise reveal themselves at high energies, the partial hypercharge gauge invariance is proposed. According to it, while the electroweak theory is basically $SU(2)\otimes U(1)_Y$ gauge invariant being constructed from ordinary covariant derivatives of all fields involved, the $U(1)_Y$ hypercharge gauge field B_μ field is allowed to form all possible polynomial couplings on its own and with other fields invariants. This unavoidably leads to the SLIV with the VEV being developed on some B field component, while its other components convert into the massless Nambu-Goldstone modes. After

standard electroweak symmetry breaking they mix with a neutral W^3 boson of $SU(2)$ that leads, as usual, to the massless photon and massive Z boson. Along with this the partial gauge invariance provides some distinctive SLIV effects in a laboratory mainly through slightly deformed dispersion relations of all SM fields involved. Being naturally suppressed at low energies these effects may become detectable in high energy physics and astrophysics. In contrast to the previous pure phenomenological studies, this theoretical approach allows to make in general the more definite predictions (or verify some earlier assumptions made ad hoc), and also discuss not only the time-like Lorentz violation but also the space-like case on which the current observational bounds appear to be much weaker.

(5) As one may note, predictions of the extended QED with non-exact gauge invariance (chapter III) and those of of the extended SM with partial gauge invariance (chapter IV) are noticeably differed. The reason behind this is that the high dimension operators for minimal extensions of SM and QED are different. While for QED we can construct dimension-5 operator $\frac{1}{\mathcal{M}}D'^{*}_{\mu}\overline{\psi}\cdot D'^{\mu}\psi$, which modifies dispersion relation only for charged fermions, for SM we need dimension-6 operator (4.42), which modifies every dispersion relation for every SM fields. Modification which is caused in QED can be invoked in SM as well with the following high dimension operator

$$\frac{B_\mu}{M_p^2}\left[iH^+\left(\overline{e}_R\overleftrightarrow{\partial^\mu}L\right)+h.c\right] \tag{5.1}$$

sufficiently suppressed by Plank mass square. Easy to see that after Lorentz and internal SU(2) symmetry breaking it will lead to the modification like in (3.6),

$$i\frac{MV}{M_p^2}n_\mu\overline{\psi}_e\overleftrightarrow{\partial^\mu}\psi_e \tag{5.2}$$

and correspondingly to the dispersion relation of type (3.7). Here M is Lorentz violation scale, while V is a electro-weak violation scale, which is expected to be much lower then Lorentz violation scale, that gives only small correction to the modification discussed in the chapter four. Despite the fact that they have same dimensions, operator (4.42) will be dominant at all scales.

Though we mainly focused here on the lowest-order extensions of SM as in B field potential terms, so in its polynomial couplings with other SM fields invariants in (4.1), our conclusions are likely to remain in force for any other extensions as well provided that the partial gauge invariance conjecture in a special form stated above for the hypercharge Abelian symmetry $U(1)_Y$ is basically satisfied. In this connection, further study of this conjecture in a more general context, particularly in general Yang-Mills theories and gravity (where again the simple potential-like extension of the theory appears insufficient to lead to an actual physical Lorentz violation [31]) seems to be extremely interesting.

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